



Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations

1. Thesis and Dissertation Collection, all items

1949

Analysis of a double skin and corrugated thin flat plate.

Irgens, Donald Leroy

University of Michigan

<http://hdl.handle.net/10945/6322>

Downloaded from NPS Archive: Calhoun



Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

<http://www.nps.edu/library>

NPS ARCHIVE
1949
IRGENS, D.

Thesis
157

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTF... 93943-5101

Library
U. S. Naval Postgraduate School
Annapolis, Md.

Minneapolis, Minnesota
July 15, 1949

Faculty Committee
Department of Aeronautical Engineering
University of Minnesota
Minneapolis, Minnesota

Sirs:

Enclosed herewith is completed thesis submitted to you for approval.

Subject thesis is for partial fulfillment of the requirements for a degree of Master of Science in Aeronautical Engineering.

RECEIVED
JULY 10 1954

RECEIVED
JULY 10 1954
JULY 10 1954
JULY 10 1954

1954

RECEIVED
JULY 10 1954
JULY 10 1954

RECEIVED
JULY 10 1954
JULY 10 1954
JULY 10 1954

3011-4A 241
 PNP
 24305

~~TABLE OF CONTENTS~~

Summary	1-2
Introduction	3
Significance and Procedure	4-5
Methods and Discussion	6-12
Conclusions	13-14
Appendix A	15-16
Appendix B	17-18
Appendix C	19-20
Appendix D	21
Appendix E	22-23

ANALYSIS OF A
 DOUBLE SKIN AND CORRUGATED
 THIN PLAT PLATE

by
 Donald L. Irgens

July 15, 1949

TABLE OF CONTENTS

Summary -----	Pages 1-2
Introduction -----	" 3
Equipment and Procedure -----	" 4-8
Results and Discussion -----	" 9-12
Conclusions -----	" 13-14
Sample Calculations -----	" 16-18
Appendix A -----	" 18-25
Bibliography -----	" 15
Diagrams, Tables and Graphs -----	" 26-50

TABLE OF CONTENTS

1-1	Summary
2	Introduction
3-4	Development and Evolution
5-6	General and Specific
7-8	General and Specific
9-10	General and Specific
11-12	General and Specific
13-14	General and Specific
15-16	General and Specific
17-18	General and Specific
19-20	General and Specific
21	Summary
22-23	General and Specific

SUMMARY

An experimental investigation was made on the behavior of a test panel constructed of 0.025 aluminum corrugation with 0.032 aluminum skin on both sides. Such a panel was subjected to a compressive load in the longitudinal direction with the unloaded edges simply supported. The panel was long in reference to its width.

Tests showed that inter-rivet buckling induced by inferior blind riveting or too large a rivet spacing would precipitate early panel failure before actual critical load would occur, on a well designed panel.

Such type of construction proved very stable up to loads approaching the critical value. The strength weight ratio indicated it to be very desirable in aircraft construction.

An analytical analysis was made using both non-isotropic and isotropic plate theory; both methods predicted the critical load to within 7.5 per cent of each other, thus indicating that the double skin and corrugation type plate, of the design under consideration, acts much like an isotropic plate.

Because of the fact that such a plate will buckle at a load that will induce strains in the sheet and corrugation above their elastic limits, it was necessary,

CONCRETE

CHAPTER 1

In concrete construction, the use of the
best of a long time period is 2.0% of the
concrete with 0.5% of the concrete in the
form and is subjected to a compressive load in the
longitudinal direction with the intended stress
applied. The steel and load in concrete is 10
times.

Tests show that the effect of the load is
increased by the effect of the load in the
concrete and the effect of the load in the
concrete is 10 times the effect of the load in the
concrete. The effect of the load in the
concrete is 10 times the effect of the load in the
concrete. The effect of the load in the
concrete is 10 times the effect of the load in the
concrete.

In concrete construction, the use of the
best of a long time period is 2.0% of the
concrete with 0.5% of the concrete in the
form and is subjected to a compressive load in the
longitudinal direction with the intended stress
applied. The steel and load in concrete is 10
times.

Tests show that the effect of the load is
increased by the effect of the load in the
concrete and the effect of the load in the
concrete is 10 times the effect of the load in the
concrete. The effect of the load in the
concrete is 10 times the effect of the load in the
concrete.

in the analytical analysis, to use reduced values for the modulus of elasticity. The prediction of the behavior of the metal in the plastic range is not exact thus indicating such a prediction as critical load is difficult for this type construction.

In the analytical study, we are required to follow the
 the method of analysis. The question of the pos-
 sibility of the study in the classic sense is not raised
 from the fact that a possibility of analysis is
 possible for this type of analysis.

The question of the possibility of analysis is not raised
 from the fact that a possibility of analysis is
 possible for this type of analysis. The question of the
 possibility of analysis is not raised from the fact that
 a possibility of analysis is possible for this type of
 analysis.

The question of the possibility of analysis is not raised
 from the fact that a possibility of analysis is possible
 for this type of analysis. The question of the possibility
 of analysis is not raised from the fact that a possibility
 of analysis is possible for this type of analysis.

The question of the possibility of analysis is not raised
 from the fact that a possibility of analysis is possible
 for this type of analysis. The question of the possibility
 of analysis is not raised from the fact that a possibility
 of analysis is possible for this type of analysis.

The question of the possibility of analysis is not raised
 from the fact that a possibility of analysis is possible
 for this type of analysis. The question of the possibility
 of analysis is not raised from the fact that a possibility
 of analysis is possible for this type of analysis.

The question of the possibility of analysis is not raised
 from the fact that a possibility of analysis is possible
 for this type of analysis. The question of the possibility
 of analysis is not raised from the fact that a possibility
 of analysis is possible for this type of analysis.

INTRODUCTION

The problem under consideration in this report was to study the behavior of a plate of the double skin corrugation construction when subjected to a compression load. Special emphasis was given to the determination of the critical buckling load.

A considerable amount of research work on this type of construction has been carried out by the Glenn L. Martin Aircraft Factory, but considering the complexities that arise in the analysis of any non-isotropic plate, additional research work on it seemed warranted.

The analysis was limited to aluminum alloy plates so designed that the initial buckling stress of the skin and the crippling stress of the corrugation were approximately equal so as to eliminate skin buckling before panel failure.

The experimental tests were limited to a plate loaded with a uniform axial load parallel to the sides of the plate; the unloaded edges were simply supported.

The thesis work was done by the writer during the school year 1948-49 at the University of Minnesota. Thesis advisor was Professor Joseph A. Wise.

DISCUSSION

The positive effect considered in this report was to study the behavior of a plate of low ductility under tension. The material was subjected to a compression load. The results are given in the following table. The critical loading load.

A considerable amount of research was on this type of material and has been carried out by the team in the last few years, and considering the complexity of the subject in the analysis of low ductility plates, additional research was on it seemed warranted.

The results are listed in the following table as designed for the critical loading stress of the plate and the existing stress of the compressed plate. The results are as for aluminum plate loading below the critical stress.

The experimental results were listed in a table located after a certain value had reached in the plate of the plate the uniaxial stress were simply supported. The results were also of the plate during the critical stress from 1945-46 at the University of Minnesota. Results section are (Table 1) and (Table 2).

EQUIPMENT AND PROCEDURE

Test Specimen.

The details of the test panel are shown on Fig. 2.

The material for the panel was furnished by the Glenn L. Martin Aircraft Factory, fabrication of the plate was by the writer.

As noted in Fig. 2 the panel is tapered in plan-form and thickness, the original plans for the investigation called for a non-tapered section but due to non-availability of desired material the plate was constructed as shown. It was felt that the small amount of taper did not materially change the conditions that would be found in a non-tapered section. For conservatism the minimum section was used for the analytical analysis.

Internal strain gauges with necessary leads were placed before fabrication of cover skins. All strain gauges were placed in duplicate at locations as shown on Fig. 2.

Filler blocks along supported sides so designed to separate skins along supports proved very successful in providing a symmetric support.

Filler blocks at top and bottom of plate designed primarily to transmit the load to the corrugation, were found unnecessary and were removed after initial test.

THE RESEARCH

The results of the first phase of the study are shown in Table 1.

The material for the second phase was obtained by the

analysis of the results of the first phase of the study.

Table 2 shows the results.

As shown in Table 3, the results of the first phase

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

showed that the results of the first phase of the study

Principal Apparatus.

The principal apparatus required for the tests was a suitable jig to support the panel simply along the two unloaded edges; a further non-functional requirement was to make the *jig* simple and light enough for one man to handle. Such a jig was designed, details of which are shown on Fig. 4.

A 400,000 pound capacity Timus Olsen testing machine was used for subject tests. The width of the swivel head of the machine corresponded to the width of the top of the panel.

Minor Apparatus.

The leads from the strain gauges were soldered into a junction box made up with Mallory infant jacks; connections from junction box to Baldwin and Southwark "K" box was by electrical phone plugs.

Procedure.

The general procedure of the tests was to load the plate by increments, then take strain and deflection readings. It was found that for the load that was placed on the plate during the tests no appreciable deflection was observed, an indication that such type construction is very stable up to critical load.

Three separate tests were made, results of which are shown graphically in Figs. 6 to 14 and in tabular form on Tables I to III.

Defining the Problem

The principal problem is to find out what the cause of the trouble is. It is not enough to know that there is a trouble, but we must know what it is. For this purpose, we must first of all define the trouble. We must know what we are looking for. We must know what the trouble is. We must know what the trouble is. We must know what the trouble is.

The first step in the process of finding out what the cause of the trouble is, is to define the trouble. We must know what we are looking for. We must know what the trouble is. We must know what the trouble is. We must know what the trouble is.

Defining the Problem

The first step in the process of finding out what the cause of the trouble is, is to define the trouble. We must know what we are looking for. We must know what the trouble is. We must know what the trouble is. We must know what the trouble is.

Defining the Problem

The first step in the process of finding out what the cause of the trouble is, is to define the trouble. We must know what we are looking for. We must know what the trouble is. We must know what the trouble is. We must know what the trouble is.

Tests are described separately below giving details of each.

Test I.

Load increments of 6,000 pounds were applied; at panel load of 23,000 pounds local failure occurred at top of panel. Upon investigation of local failure it was found that edge supports did not run up high enough, giving an unsupported section of 2 inches at top of plate. Corrective measure taken was to remove the upper $1\frac{1}{2}$ inches of plate, giving only a $\frac{1}{2}$ inch unsupported plate for test number two.

Test II.

Load increments of 6,000 pounds were applied with one intermediate panel load of 19,850 pounds. Panel load went up to 30,000 pounds at which point local failure occurred at bottom of panel. Upon investigation of local failure it was found that the filler block on bottom did not transmit the load properly to the corrugation and caused a local failure in bearing of the corrugation. This was corrected by trimming $2\frac{1}{2}$ inches from the bottom and removing filler block.

Test III.

Load increments of 12,000 pounds were applied up to 24,000 pounds, plate remained stable up to 30,000

These are important questions which arise in the
 of the

the

the

the

the

the

the

the

pounds at which point readings were recorded. Upon increasing the load to 31,600 pounds a failure occurred at approximately 12 inches from the top of the panel. The failure at first appeared to be the critical load for the panel but upon examination of failure it was observed that at point of failure, on one of the cover sheets, one horizontal row of rivets had pulled out thus subjecting the skin to twice its original unsupported length, this reduced the critical buckling stress of the sheet to a point which precipitated the failure.

Above plate load of 30,000 pounds it was observed that inter-rivet buckling had started.

Upon reaching the load where one row of rivets pulled out it appeared as though plate had started to form 3 definite half waves of approximately equal length, the loading was not continued however since another test was to be run on the undamaged remainder.

Test IV.

Plate was trimmed to a 56 inch length. Load increments were applied similar to those in test three.

Inter-rivet buckling was observed to start at panel load of 31,000 pounds. One definite half wave formed in the longitudinal direction at a load of 34,000 pounds at which point severe inter-rivet buckling occurred on one plate, while other plate remained flat. Maximum deflection of half wave was at center of plate and was estimated

to be one and one half inches, since deflection was greater than Ames dial capacity at that point.

10. De ontz and een half literen: kleine veldwijzen

Address: 1000 1st St. N. Minneapolis, MN 55401

RESULTS AND DISCUSSION

Load, strain data is presented on Tables I-IV. The principal strains were obtained from rosette readings by method described in Reference 2 of bibliography. Plots of principal stresses versus plate loads are shown on Figs. 5-14.

It was unfortunate that none of the four tests produced a true critical load on the plate, however, it was felt that the tests proved successful as they clearly showed the stability of this type construction in the region of design load. The indications of the tests were such that the panel would be well in its plastic range before the critical load would be reached; a theoretical analysis, which will be presented subsequently, bears this fact out quite clearly.

Inasmuch as the critical load on a panel such as this is of no practical value, it is used extensively as a design criteria, that is by being able to predict what the critical load is and knowing the behavior of the plate up to critical, a safe design load may be arrived at.

The tests bore out the fact that any rivet failure will induce early skin instability, it appeared that had the rivet spacing been decreased to three fourths of an inch, the skin would have remained stable and

Local, street cars in operation on October 2-17.
The principal stations were situated from 100 to 200
feet by means described in reference 1 of this report.
Plans of principal stations shown (see page 101)
shown on page 2-10.

It was determined that some of the first cars
operated a few stations back on the line, however,
it was felt that the first car was necessary in that
it was shown the results of this type construction
in the region of the line. The location of the
first car was from the point where it left the
main road before the station line was in service;
a standard design, which will be referred to later
as the "standard" design.

However, as the station line on a road was in
this is of no practical value, it is not necessary
as a station design, but it is being used in practice
that the station line is not showing the results of
the work up to date, a new design had not yet
arrived at.

The station line and the first car were taken
with some very little difficulty, it appeared that
the first car was being taken from the station
at the first few miles from the station line.

probably would have allowed the plate to reach a full critical value.

Theoretical Analysis.

The method used in the theoretical analysis follows quite closely the work as presented in Reference 1.

The critical load on a non-isotropic plate similarly loaded and supported to the one of this test is given in Reference 3 as:

$$P_{CR} = \frac{2\pi^2}{b^2} \sqrt{(D_x)(D_y) + D_3}$$

where: P_{CR} = critical load in pounds per unit width
 b = plate width, small in reference to length
 D_x = average flexural rigidity per unit width of the plate parallel to the y axis
 D_y = same as D_x but parallel to x axis
 $D_3 = \frac{1}{2} [\mu D_y + \mu D_x] + 2(GI)_{xy}$
 $2(GI)_{xy}$ = average torsional rigidity of the plate (ratio of the moment per unit width to the twist)

$$\text{also: } D_x = \frac{(EI)_x}{1 - \mu_{xy}} , D_y = \frac{(EI)_y}{1 - \mu_{xy}}$$

Since flexural rigidity about an axis parallel to y is contributed to by both sheet and corrugation, and about an axis parallel to x is contributed mainly by sheet, D_x and D_y are conservatively written as:

provisionally with regard to the value of the α parameter.

Generalization

The method used in the preceding chapter for the determination of the α parameter is based on the assumption that the α parameter is a function of the β parameter. This assumption is not valid in general, but it is valid for the case of a linear relationship between the α and β parameters.

$$\alpha = \frac{\beta}{\beta + 1} \sqrt{\frac{1}{2} \ln 2}$$

It is assumed that the α parameter is a function of the β parameter. This assumption is not valid in general, but it is valid for the case of a linear relationship between the α and β parameters. The α parameter is a function of the β parameter, and the β parameter is a function of the α parameter.

$$\alpha = \frac{\beta}{\beta + 1} \sqrt{\frac{1}{2} \ln 2}$$

The α parameter is a function of the β parameter, and the β parameter is a function of the α parameter. The α parameter is a function of the β parameter, and the β parameter is a function of the α parameter.

$$\alpha = \frac{\beta}{\beta + 1} \sqrt{\frac{1}{2} \ln 2}$$

Since the α parameter is a function of the β parameter, and the β parameter is a function of the α parameter, the α parameter is a function of the β parameter, and the β parameter is a function of the α parameter.

$$D_x = \frac{(E_{sheet} I_{sheet})_x}{1 - \mu^2} + (E_{corr} I_{corr})_x$$

$$D_y = \frac{(E_s I_s)_x}{1 - \mu^2}$$

For further conservatism effect of corrugation in D_x is omitted in evaluation of D_3 , hence:

$$D_3 = \frac{(E_s I_s)_x}{1 - \mu^2} + 2(GI)_{xy}$$

In Reference 1 it is shown that the term $2(GI)_{xy}$ is a constant and can be replaced by $G_e t_s d^2$, where:

G_e = effective shear modulus

t_s = bare skin thickness

d = distance between skins

Tests conducted on corrugation columns at the Glenn L. Martin Company indicate that the tangent modulus of elasticity of the compression stress strain curve can be used to predict the value of E_c for the type plate in question, hence Fig. 18 was plotted from properties as given in Fig. 17.

$(E_s)_x$ is also taken as the tangent modulus of the average strain strain curve from the properties of flat sheet when the critical buckling load has been exceeded. Such a curve is plotted on Fig. 20 from data of Fig. 19. For conservatism $(E_s)_y$ has been taken as equal to $(E_s)_x$.

Effective shear modulus has been studied quite extensively for this type construction in Reference 1,

$$e^{j\omega t} \cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2} = \cos(\omega t)$$

$$\frac{d(\ln \sigma_{\text{eff}})}{d \ln \omega} = 2$$

was included in evaluation of 24 houses

$$\lim_{x \rightarrow 0} \frac{x \ln(x+1)}{x^2} = \frac{1}{2}$$

in the literature. It is shown that the time $\mathcal{E}(n)$ to

Was ist das Ziel der militärischen Ausbildung?

Copyright © 1999 by John Wiley & Sons, Inc.

...the ... of the ...

Copyright © 2003 by John Wiley & Sons, Inc.

from 1946 to 1950, the number of cases of tuberculosis in the United States was 1,000,000.

2002 The Authors
Journal compilation © 2002 Blackwell Science Ltd

1433 To address this issue, the authors have

Abstracts of the following papers were presented:

Cost a nurse is listed as \$12.00 from date of 7/1/00.

¹² Large as noted with $\gamma(g)$ unknown.

1991

BYE-WORD: every morning and some special calls

Continued on next page

and a value of 0.7 G has proved to be quite reliable for cases of the plate being stressed above the elastic limit.

By assuming a series of maximum strains and calculating P_{CR} by the non-isotropic plate equation just given, then finding corresponding load a stable plate will carry at similar strains by $P'_{CR} = \sigma_{corr} A_{corr} + \sigma_{sheet} A_{sheet}$ and plotting results, the intersection of two plots will give predicted critical load for panel; such a plot is given on Fig. 21. A typical calculation for making the graph is given under Sample Calculations of this thesis.

The predicted critical load was determined to be 2,900 pounds per inch of width; for an average width of 17 inches between supports this would correspond to a total panel load of 49,300 pounds. Maximum strain in plate at critical load was predicted at 0.00345 inches per inch.

This theoretical analysis indicates the plate goes well into the plastic range of the material before critical load is obtained. In reference to actual test results it can be readily seen that plate was obeying Hooke's Law almost up to the point where local skin buckling precipitated panel failure, and that if skin had not buckled prematurely load could have continued up to near predicted critical.

and a letter of G.I. 2 was received by the same person
for reason of the same being received from the same
person.

[illegible][illegible]

and not detailed personally and with discretion
providing necessary means to follow, and that it was
made's law aimed up to the point where power was
vested it can be readily seen that there was decided
evidence that is abundant. In reference to actual law
will into the public mind of the country and
this Government's activities and the right power

CONCLUSIONS

Corrugation and double skin type of construction forms a very stable panel and maintains this stability very close to its critical load. The strength weight ratio is very high, a factor of significance in aircraft construction.

Relating the test results to theoretical calculations indicates that a design load critical load ratio is in the neighborhood of 0.6.

The need for precise riveting and rigid inspection after completing a panel seems very necessary since panels of this type depend largely on the flexural rigidity contributed by the sheets, for stability.

The test results did not indicate a true critical load, however had inter-rivet buckling been suppressed by a more precise rivet job or a decreased rivet spacing it is believed the actual and predicted critical load would have approached each other.

The analytical analysis is by no means an exact one, since various assumptions as to the behavior of the plate in the plastic range have been made, however it appears quite reliable in reference to repeated applications by the Glenn L. Martin Aircraft Company.

The plate apparently can be treated as an isotropic plate based on the analysis of an isotropic plate by

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific information required.

1. The first step in the process of identifying a problem is to define the problem. This involves identifying the symptoms of the problem and determining the scope of the problem. Once the problem has been defined, the next step is to identify the causes of the problem. This involves identifying the factors that are contributing to the problem and determining the underlying causes. Once the causes have been identified, the next step is to develop a plan of action. This involves identifying the steps that need to be taken to solve the problem and determining the resources that will be needed to implement the plan. Finally, the last step in the process is to implement the plan and monitor the results. This involves putting the plan into action and tracking the progress of the solution. Once the problem has been solved, the final step is to evaluate the results and determine if the solution was effective. This involves comparing the results of the solution to the original problem and determining if the problem has been solved. If the problem has not been solved, the process may need to be repeated.

THE HOUSE FOR SPECIAL STUDIES AND RESEARCH

1. The first point is that the Commission has not yet received any information from the Government of the Republic of China (Taiwan) regarding the situation in the region.

It is believed the above was provided without issue.

The following is a list of the names of the persons who have been appointed to the various positions in the various departments of the Government of the State of New York, for the year 1900.

The above agreement was provided as an exhibit to the letter agreement by the above named company.

11

the method of strain energy, as given in Appendix A. The analysis as given in Appendix A predicts a critical load about 7.5 per cent higher than by analyzing the plate as being non-isotropic.

The method of which we speak is known as
the method of the *l'Algebra* & *l'Algebra* is a method
found about 1700 but not before that by which the
this is an *Algebra* & *Algebra*.

The method of the *l'Algebra* & *Algebra* is a method

found about 1700 but not before that by which the

this is an *Algebra* & *Algebra*.

The method of the *l'Algebra* & *Algebra* is a method

found about 1700 but not before that by which the

this is an *Algebra* & *Algebra*.

The method of the *l'Algebra* & *Algebra* is a method

found about 1700 but not before that by which the

this is an *Algebra* & *Algebra*.

The method of the *l'Algebra* & *Algebra* is a method

found about 1700 but not before that by which the

this is an *Algebra* & *Algebra*.

The method of the *l'Algebra* & *Algebra* is a method

found about 1700 but not before that by which the

this is an *Algebra* & *Algebra*.

The method of the *l'Algebra* & *Algebra* is a method

found about 1700 but not before that by which the

this is an *Algebra* & *Algebra*.

The method of the *l'Algebra* & *Algebra* is a method

found about 1700 but not before that by which the

this is an *Algebra* & *Algebra*.

BIBLIOGRAPHY

1. Kennedy, W. B., Troxell, W. W.: Study of Compression Panel Supported on Four Edges, Formed of Corrugated Sheet with Flat Skin on Both Sides. Structural Development Report 21, Glenn L. Martin Company, 1944.
2. Wise, Joseph A.: Circles of Strain. Journal of the Aeronautical Sciences, Vol. 7, No. 10, pp. 433-40, August 1940.
3. Timoshenko, S.: Theory of Elastic Stability, McGraw-Hill Book Company, 1936.

EXHIBIT 124

1. The first of the two items is a letterhead from the
American Red Cross, dated 1941, and is addressed to
the American Red Cross, 1215 North 17th Street, New York,
New York. The letter is signed by the American Red Cross,
New York, and is dated 1941. The letter is addressed to
the American Red Cross, 1215 North 17th Street, New York,
New York.

2. The second of the two items is a letterhead from the
American Red Cross, dated 1941, and is addressed to
the American Red Cross, 1215 North 17th Street, New York,
New York. The letter is signed by the American Red Cross,
New York, and is dated 1941. The letter is addressed to
the American Red Cross, 1215 North 17th Street, New York,
New York.

3. The third of the two items is a letterhead from the
American Red Cross, dated 1941, and is addressed to
the American Red Cross, 1215 North 17th Street, New York,
New York. The letter is signed by the American Red Cross,
New York, and is dated 1941. The letter is addressed to
the American Red Cross, 1215 North 17th Street, New York,
New York.

SAMPLE CALCULATIONS

For: $P_{cr} = \frac{2\pi^2}{b^2} \left[10 D_1 (E_s) + D_3 \right] \quad \text{Per. page Fig. 21}$

$$P_{cr} = \sigma_{corr} A_{corr} + \sigma_{shear} (A_{ve.}) A_{sheet}$$

From: $E_s = 29,000 \text{ ksi}$

From: $G_s = 0.4 E_s = 11,600 \text{ ksi}$

$F = 10^6$ below ultimate limit

$I_s = 0.005475 \text{ in.}^4/\text{in.}, H_s = 0.004 \text{ in./in.}$

$I_c = 0.00159 \text{ in.}^4/\text{in.}, H_c = 0.0015 \text{ in./in.}$

$d = 6.25 \text{ in.}$ min. distance between skins

$t_s = 0.0272 \text{ in.}$ (bare 0.0324 in. Al. min)

$t_c = 0.022 \text{ in.}$ (bare 0.025 Al. min)

$b = 15.6 \text{ in.}$ { max. width of plate }

$u = 0.3 \quad (1 - u^2) = 0.91$

$\frac{2\pi^2}{b^2} = 0.00716, E_s = 29,000 \text{ ksi, } E_c = 11,600 \text{ ksi, } I_s = 0.005475$

$G t_s = 11,600 \times 0.0272 = 314.5, \sigma_{corr} = 24,000 \text{ psi, } \sigma_{shear} = 20,000 \text{ psi}$

$\therefore D_1 = \frac{1.6 \times 10^6 \times 0.005475}{0.91} + 7.4 \times 10^6 \times 0.0015$

$= 10,250 + 11,150 = 21,400$

$D_2 = 10,250 \text{ in.}, D_3 = 10(10,250) + 0.1(10,250) = 31,450$

$P_{cr} = 0.00716 / 10^6 [10,250(29,000) + 31,450]$

$P_{cr} = 24,000 \text{ psi/in.}$

20.

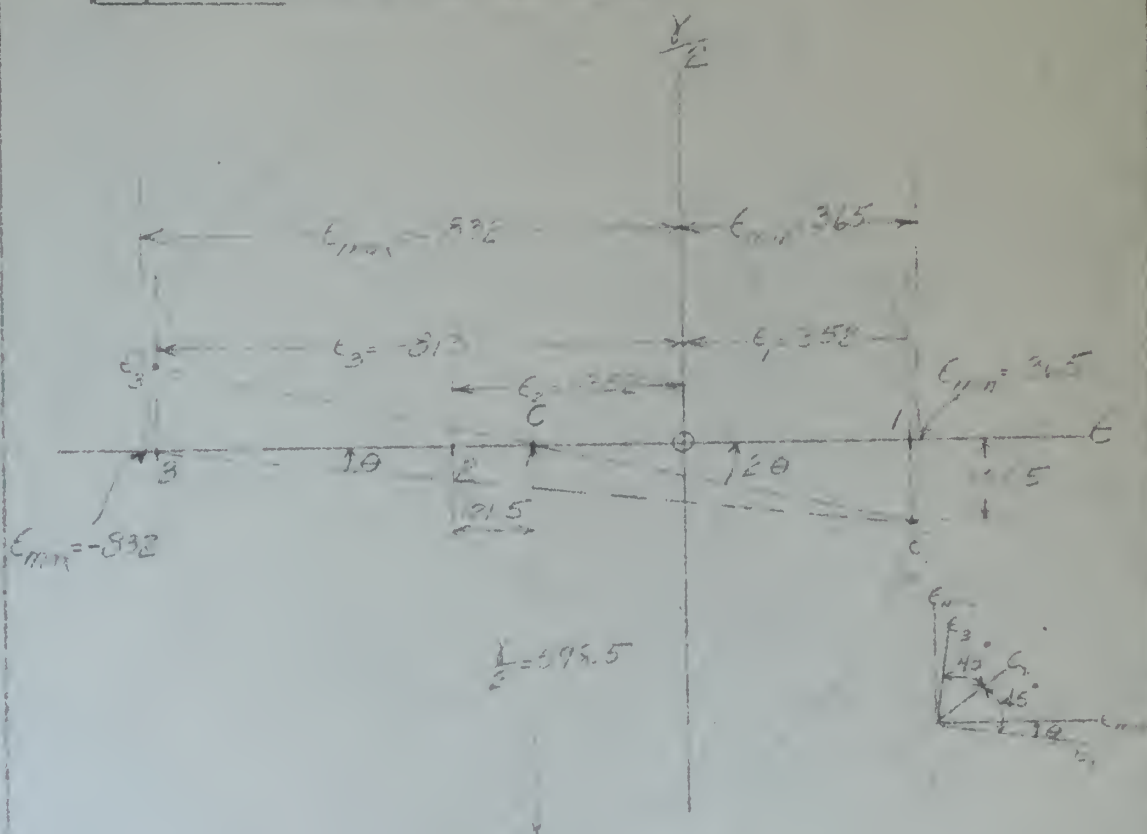
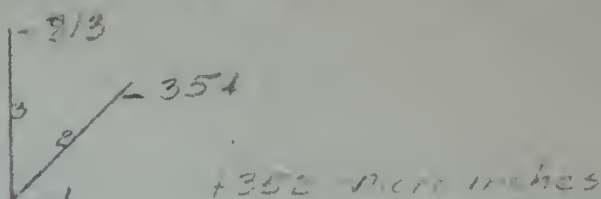
$P_{cr} = (32,500)(0.375) + (24,000)(0.64) = 31,380 \text{ lbs/in.}$

SAMPLE CALCULATIONS (Con²)

Reduction of 45° strain rosette data to principal strains. (Ref. 2)

Test #1 Load = 13,415# Table #I

Sheet #1 45° Rosette 3810 Grain top.



C is mid point between 1 and 3
 and is center of circle = 252

$$\sigma_{max} = \frac{E}{1-\mu^2} [\epsilon_{min} - \mu \epsilon_{max}], \quad \sigma_{min} = \frac{E}{1-\mu^2} [\epsilon_{min} + \mu \epsilon_{max}]$$

Figure 11

displacement at surface = zero implies
 no net surface force on side

Strain energy U is related to strain energy
 density u by $U = \int_V u \, dV$

$$U = \int_V \frac{1}{2} \sigma \epsilon \, dV$$

By assuming a deflection curve, an expression expressing the
 total strain energy can be worked out.

- Assume a deflection curve and
 configuration of isotropic plate to
 be as shown below:

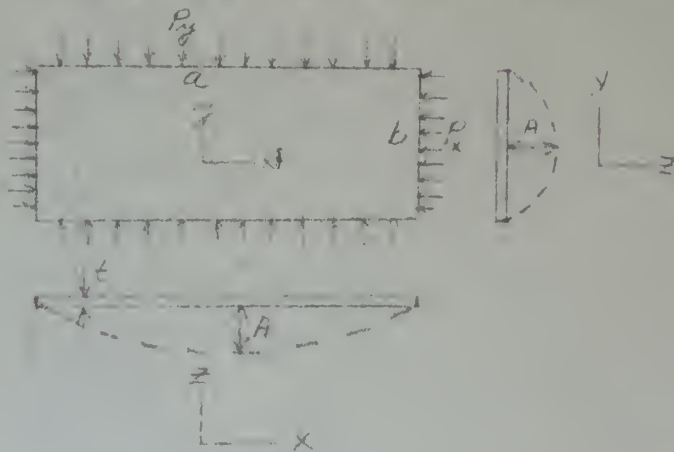


Figure 11.1

$$p_y = 45 \text{ N/m}^2 \sin \frac{\pi y}{b}$$

Where: 45 max. val.

No. Number of half
 sin waves in x dir.

No. Number of half
 sin waves in y dir.

(Total waves = 1)

By expressing stresses and strains in
 terms of deflection curve we get:

$$\sigma_x = \frac{E}{1-\mu^2} \left[\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right]$$

$$\epsilon_x = \frac{\partial^2 w}{\partial x^2}$$

$$\epsilon = \frac{E}{1.44} z \left[\frac{\partial^2}{\partial x^2} + 4 \frac{\partial^2}{\partial y^2} \right]$$

$$\epsilon = \frac{E}{1.44} z \frac{\partial^2}{\partial y^2}$$

$$\tau'_{xy} = \epsilon \gamma'_{xy} = \frac{E}{2(1+\mu)} \gamma \pm \frac{\partial^2}{\partial x \partial y}$$

$$\gamma'_{xy} = \gamma \pm \frac{\partial^2}{\partial x \partial y}$$

Expression for internal strain energy
 U_i:

$$U_i = \frac{1}{2} \int_V \left\{ \frac{E}{1.44} \left(\frac{\partial^2}{\partial x^2} + 4 \frac{\partial^2}{\partial y^2} \right) + \left(\frac{\partial^2}{\partial x^2} + 4 \frac{\partial^2}{\partial y^2} \right) \frac{\partial^2}{\partial x^2} \right\} + \frac{2E}{2(1+\mu)} \left(\frac{\partial^2}{\partial x \partial y} \right)^2 \right\} dV$$

Since $\int_V dV$ is constant and equal to I
 the I expression reduces to:

$$U_i = \frac{E I}{2(1+\mu^2)} \left\{ \left(\frac{\partial^2}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2}{\partial x^2} + 4 \frac{\partial^2}{\partial y^2} \right) \frac{\partial^2}{\partial x^2} + 4 \left(\frac{\partial^2}{\partial x^2} + 4 \frac{\partial^2}{\partial y^2} \right) \frac{\partial^2}{\partial x \partial y} \right\} dV$$

For expression in internal strain energy

$U_e = \frac{1}{2} P \delta$ where δ is displacement
 due to load P.

$$\therefore \delta = L_{max} - L_{orig}$$

$$\therefore U_e = \frac{1}{2} P (L_{max} - L_{orig})$$

By applying the binomial theorem

$$L_c = b \left[1 + \frac{a}{2} \left(\frac{dy}{dx} \right)^2 + \dots \right]$$

$$\therefore \Delta L = L_c - L_{as} = \frac{a}{2} \left(\frac{dy}{dx} \right)^2$$

$$\therefore \mathcal{U}_c = \frac{1}{2} \int \rho \left(\frac{\Delta L}{L_{as}} \right) dx$$

For given mass

$$\int \left(\frac{dy}{dx} \right)^2 dx = \frac{m^2 r^2 \rho^2 b}{4 \mu}$$

$$\int \left(\frac{dy}{dx} \right)^2 dx = \frac{m^2 r^2 \rho^2 b}{4 \mu}$$

$$\int \left(\frac{dy}{dx} \right)^2 dx = \frac{m^2 r^2 \rho^2 b}{4 \mu}$$

$$\int \left(\frac{dy}{dx} \right)^2 dx = \frac{m^2 r^2 \rho^2 b}{4 \mu}$$

$$\therefore \mathcal{U}_c = \frac{EI}{2(1-\mu^2)} \left[\frac{m^2 r^2 \rho^2 b}{4 \mu} \right] \left[m^2 b^2 + n^2 a^2 + 2m(m^2 r^2 a^2 b^2) + \dots \right]$$

$$\text{or } \mathcal{U}_c = \frac{EI}{2(1-\mu^2)} \left[\frac{m^2 r^2 \rho^2 b}{4 \mu} \right] \left[(m^2 b^2 + n^2 a^2) \right]$$

$$\text{and } \mathcal{U}_e = \int \frac{1}{2} \rho \left(\frac{m^2 r^2 \rho^2 b}{4 \mu} \right) \left(\frac{m^2 b^2 + n^2 a^2}{L_{as}} \right) dx + \int \frac{1}{2} \rho \left(\frac{m^2 r^2 \rho^2 b}{4 \mu} \right) \left(\frac{m^2 b^2 + n^2 a^2}{L_{as}} \right) dx$$

$$\text{or } \mathcal{U}_2 = \frac{1}{2} A \pi^2 \left[\frac{P_x m^2 b^2}{4a} + \frac{P_y n^2 a^2}{4b} \right]$$

then equate $\mathcal{U}_1 = \mathcal{U}_2$ For equilibrium of plate, this gives:

$$[P_x m^2 b^2 + P_y n^2 a^2] = \frac{E I \pi^2}{(1-\mu^2) a^2 b^2} (m^2 b^2 + n^2 a^2)^2$$

For Conditions at Test $P_y = 0$

Since the device skin and corrugation type plate will not buckle before reaching the elastic limit of the material it is necessary to use a reduced modulus in predicting a critical load. The method used in deriving an effective value of E in the non-isotropic analysis will be used here.

By assuming various combinations of half waves in above developed equations the minimum P loading value obtained will be the critical loading of the plate. The wave formation in the short dimension for such a case can be proven to be stable in a $\frac{1}{2}$ wave.

These bars are in equivalent stress
plate and column take the form
of five or ten bars along a
the long or short (one or short
direction)

If a bar is maximum as was
done in the last paper, then a
stress per bar would be

$$P_{bar} = \frac{P}{n} = \frac{P}{5} = \frac{P}{10}$$

For a given condition

$$P_{bar} = (30,000)(0.075) + (20,000)(0.04) = 3120 \text{ lb.}$$

For any given plate the procedure
should be to assume various maximum
stresses at which plate will reach
critical load, or then critical stresses
obtain Effective Modulus of Elasticity
of Compression and shear and
corresponding 'stresses' of the
materials at assumed stresses. Set
up results similarly to that of
Fig. 21 which will produce critical
load of plate.

From above isotropic analysis the

Predicted Critical load for plate was 3120 lbf/in. whereas non-isotropic plate theory gave a value of 2900 pounds per inch in width.

The difference is relatively small which indicates the plate acts much as an isotropic plate.

The slightly higher value obtained using the isotropic plate theory was mainly due to the fact that the flexural rigidity of the plate was due to both the sheet and corrugation about both the x and y axis, whereas in the non-isotropic theory, the contribution of the corrugation to the flexural rigidity about an axis parallel to the corrugation was omitted.

Fig. 2
Diagram of Test Panel

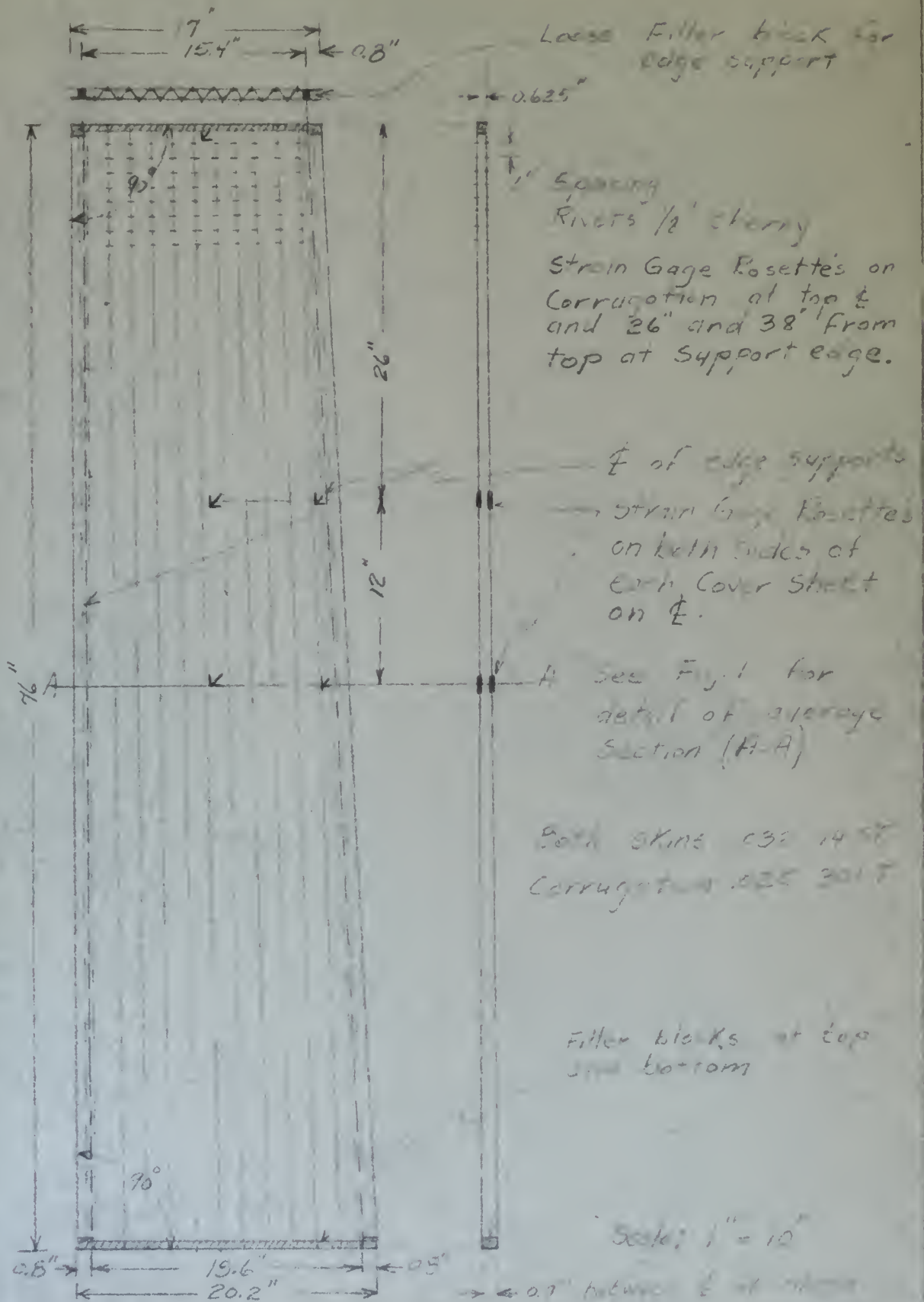


Fig. 3

Panel Loading Diagram



Note

Edges simply supporting
for support detail see
Fig. 4.

Uniform temperature
load in x direction.
Zero load in y direction.
Zero load in z direction.
 x dimension is large with
respect to y .

Fig. 4
Diagram of Testing Jig

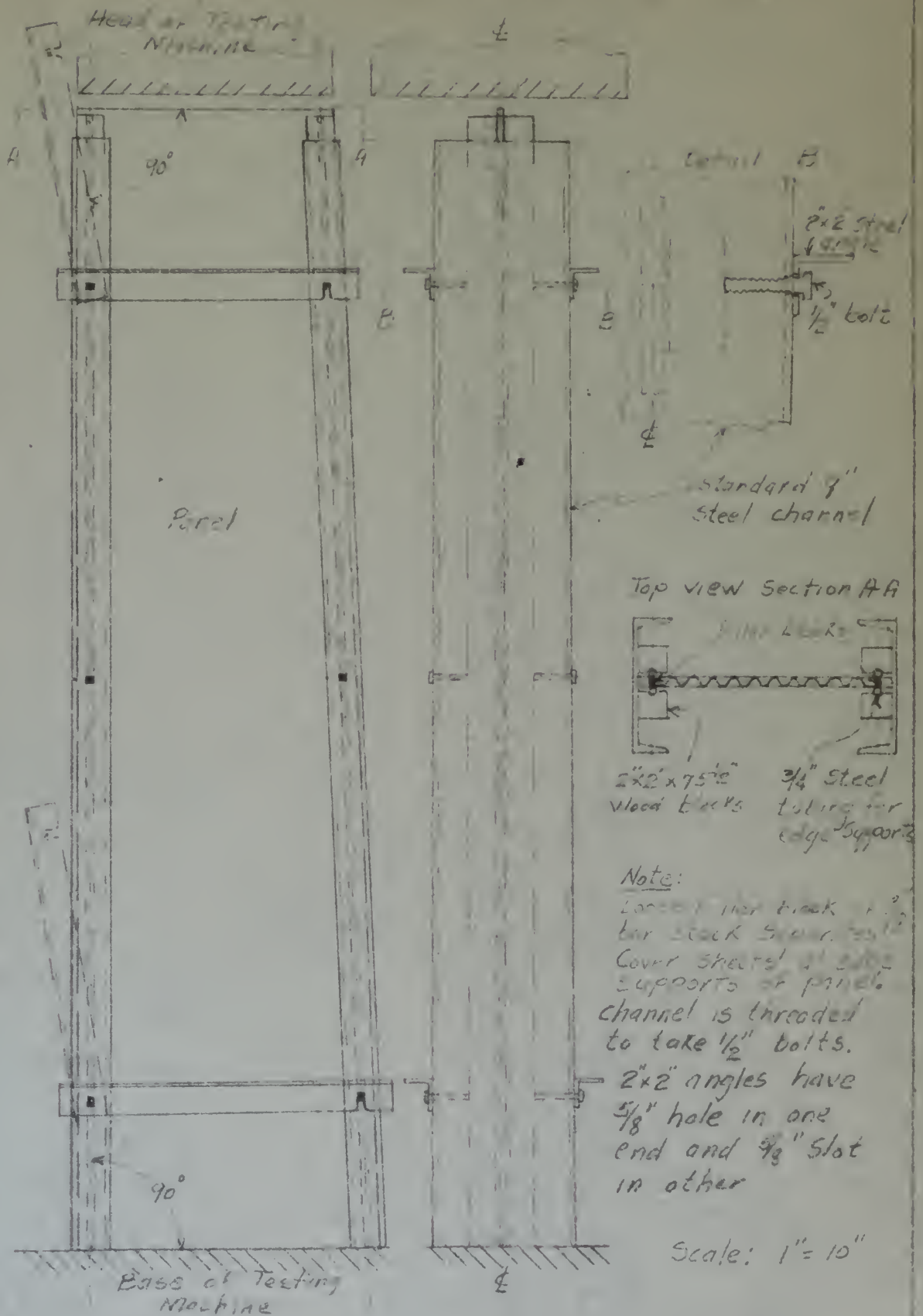


Table I
Test #1

4-13-57
Plotted on Figs 5-6-7

		Principal Load 6,000 lbs.			Principal Load 12,000 lbs.			Principal Load 18,000 lbs.		
Gage location	Gage No.	Strain in./in.	Stress lb./sq. in.	Strain in./in.	Stress lb./sq. in.	Strain in./in.	Stress lb./sq. in.	Strain in./in.	Stress lb./sq. in.	
Plate 1										
28 in. from top on edge	18									
" Horiz	19	-281		-540		-913				
" 45°	20	+121		+250		+352				
Principal		-132		-235		-354				
		240	-240	500	-505	-832	735			
		+130	+473	616	+260	+1045	215	+835	+1270	
Plate 2										
20 in. from top on edge	1-8									
" Horiz	9	-260		-544		-753				
" 45°	10	+79		+132		+204				
Principal		-131		-235		-257				
		-265	-2635	-506	-5120	-762	-7670			
		+85	+60	1347	+135	-185	2467	+214	-161	
36 in. from top on edge	11									
" Horiz	12	-235		-490		-741				
" 45°	14	+56		+162		+262				
Principal		-135		-240		-340				
		-243	-2405	-491	-4830	-750	-7350			
		+63	-110	1177	+172	+272	2551	+270	+605	
Corrugation										
26 in. from top on edge	31-44									
" Horiz	45	-96		-360		-574				
" 45°	46	+61		+132		+191				
Principal		-65		-130		-200				
		-109	-715	-362	-3536	-545	-5350			
		+75	+465	708	+134	+279	1907	+142	+313	
38 in. from top on edge	45-47									
" Horiz	48	-208		-466		-611				
" 45°	49-50	+85		+159		+235				
Principal		-74		-151		-245				
		-211	-2030	-467	-3450	-615	-5970			
		+88	+272	1151	+160	-717	2184	+240	+605	

Plate 1									
36" in from top	16								
on &	Vert	25	-624	1278			-1695		
"	Horiz	20	+270	+516			+600		
"	45°	28	-230	-520			-840		
Principal			-632 -6040	-1290 -12430			-1740 -17600		
			+274 +927 3484	+523 +1494 6762			+640 +1297 9148		

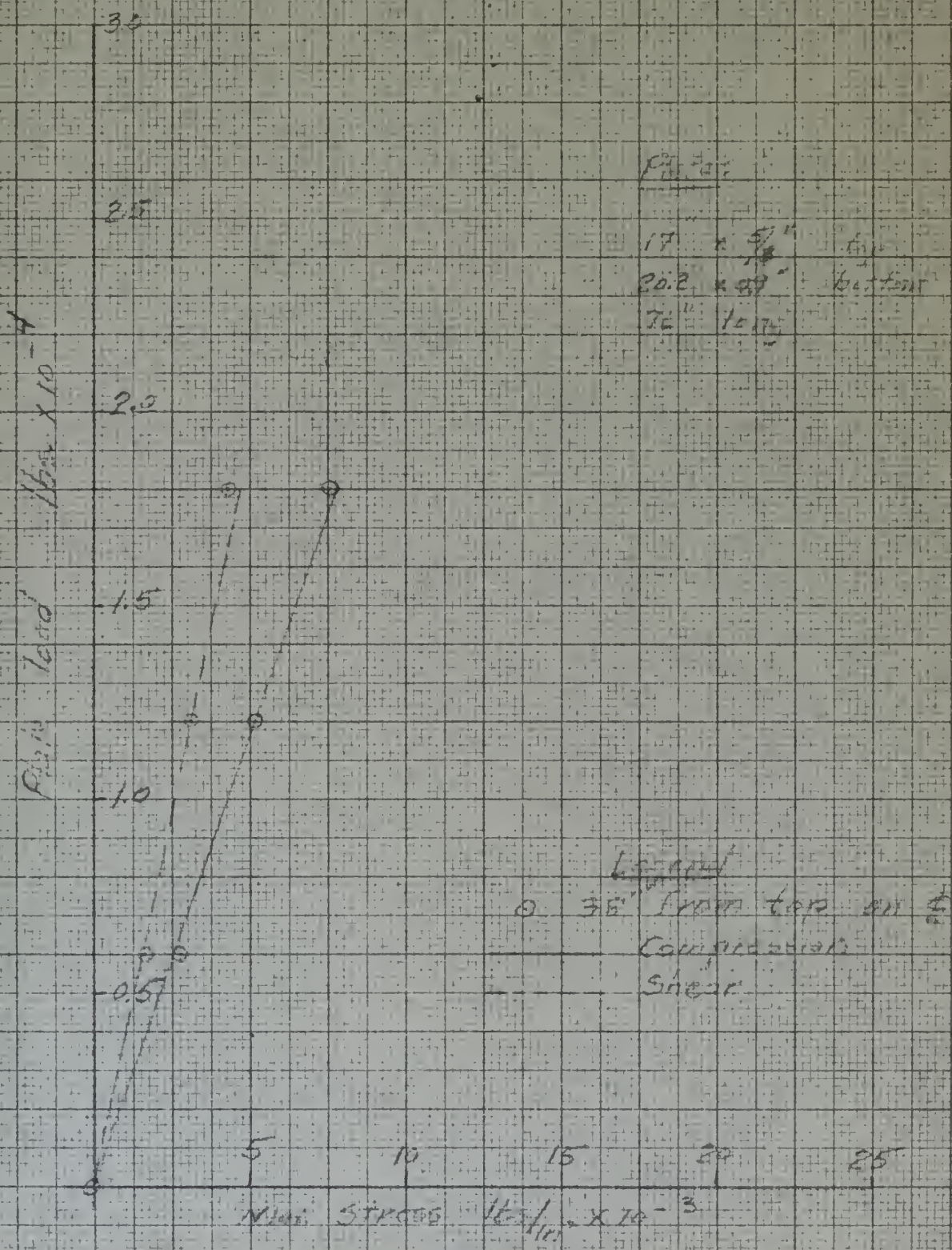
Plate 2									
36 1/2" in from top	19								
on &	Vert	9	-494	-930			-1015		
"	Horiz	9	+202	+410			+540		
"	45°	10	-175	-350			-330		
Principal			-500 -4820	-940 -8950			-1023 -9430		
			+205 +605 2762	+420 +1517 5338			+550 +2560 5995		

36 1/2" from top	4								
on &	Vert	11	-475	-785					
"	Horiz	10	+206	+455					
"	45°	14	-178	-399					
Principal			-480 -4570	-1000 -9575			-1180 -10910		
			+213 +751 2664	+461 +1770 5672			+631 +3045 6978		

Corrugation									
36 1/2" from top	40								
on edge	Vert	47	-519	1040			-1320		
"	Horiz	48	+175	+310			+480		
"	45°	50	-135	-340			-440		
Principal			-522 -5150	-1042 -10430			-1321 -12820		
			+177 +224 2587	+311 -44 5198			+481 +930 6276		

Fig. 5

Plate Load vs. Max. Cover Sheet (1) Stress
(Test #1)
See Table I



Plate

17" x 5" top
20.2" x 29" bottom
7.6" x 10.75"

Fig 16

Plate Load vs. Max. Cover Sheet (2) Stress

Test #11
Ref Table I

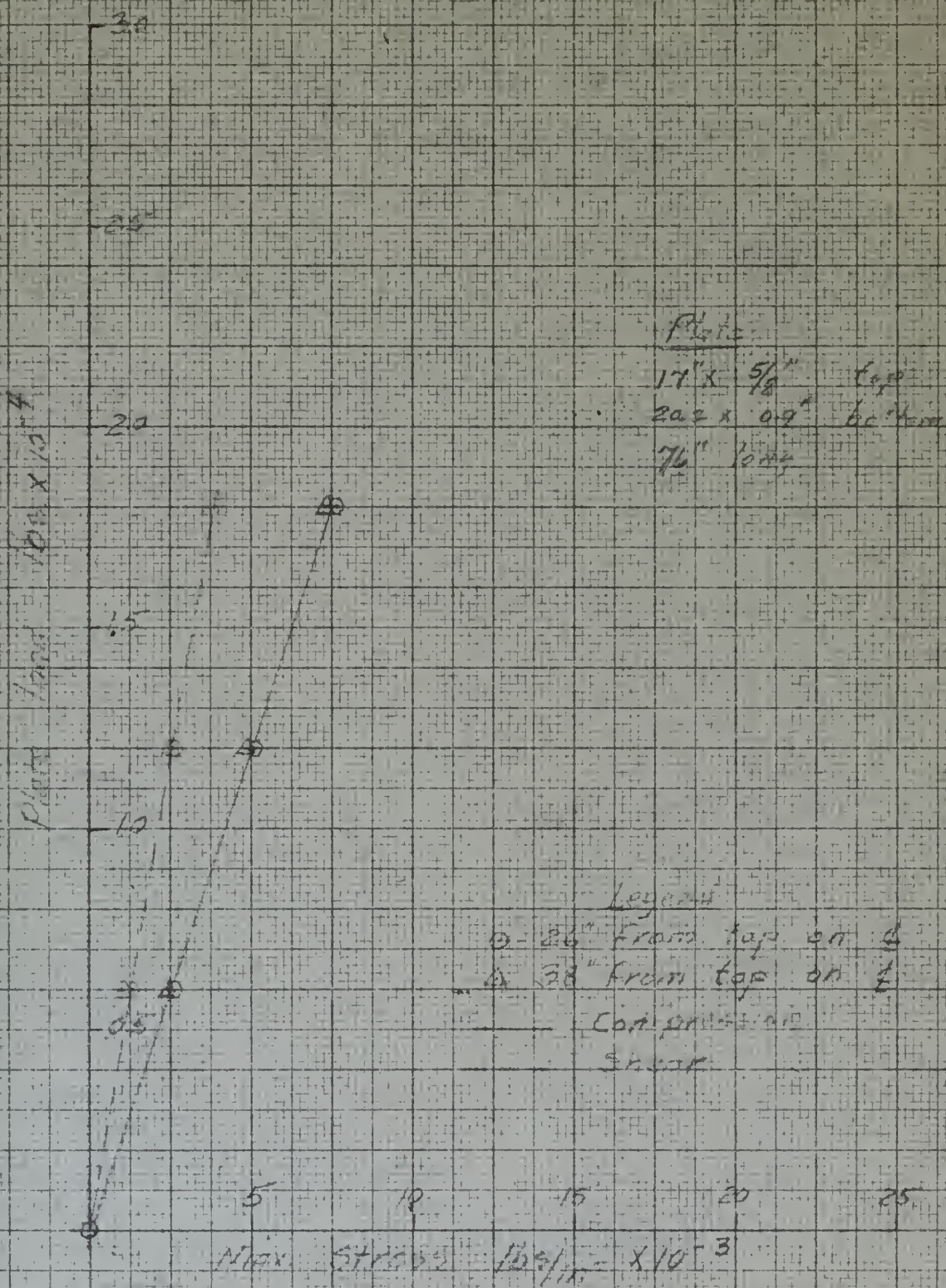


Fig. 7

Plate Load vs. Max Corrugation Stress
(Test #1)
Ref. Table I

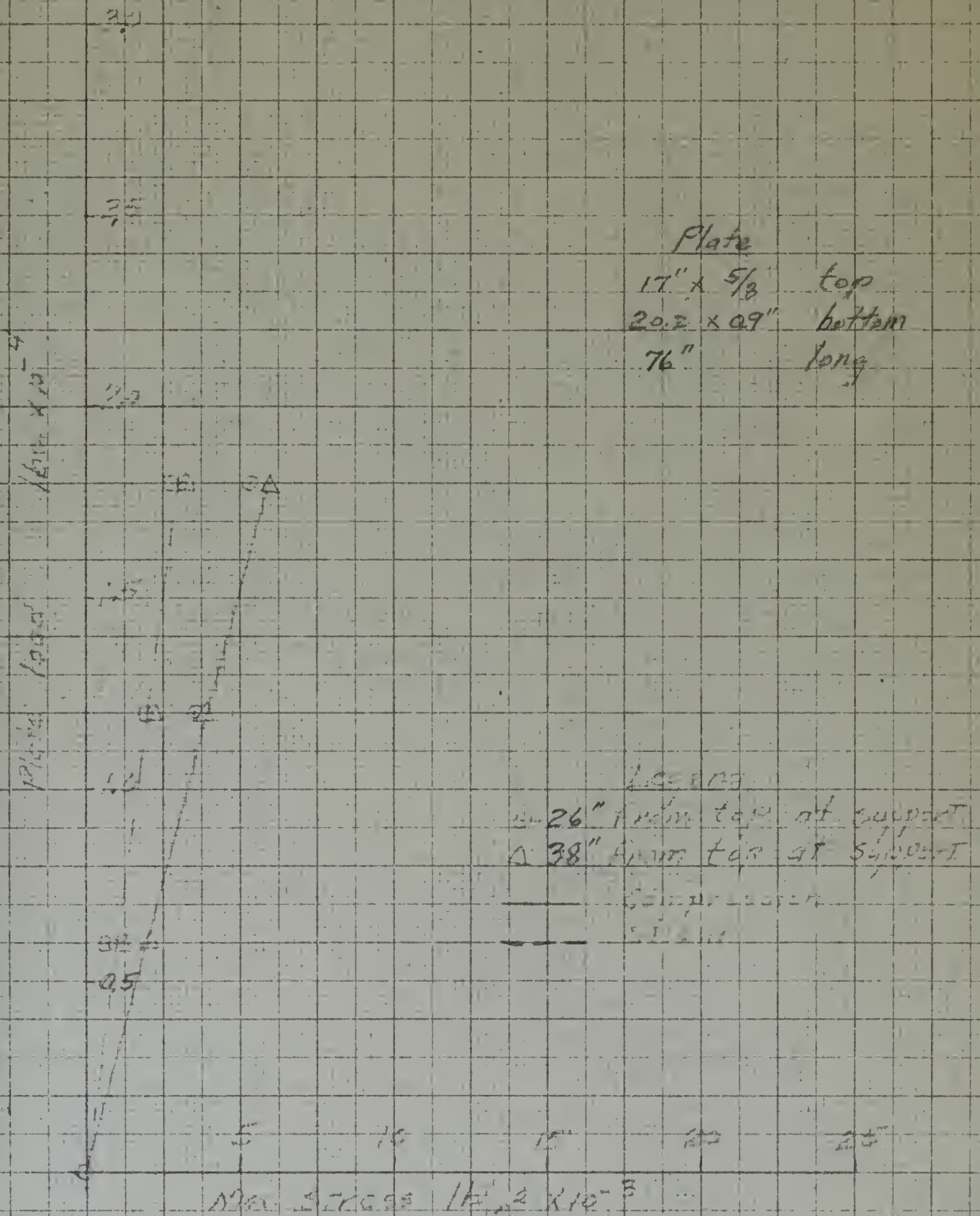


Fig. 1

Plate Load vs. Max. Comp. Stress (1) Stress
(Test #2)

Ref Table II

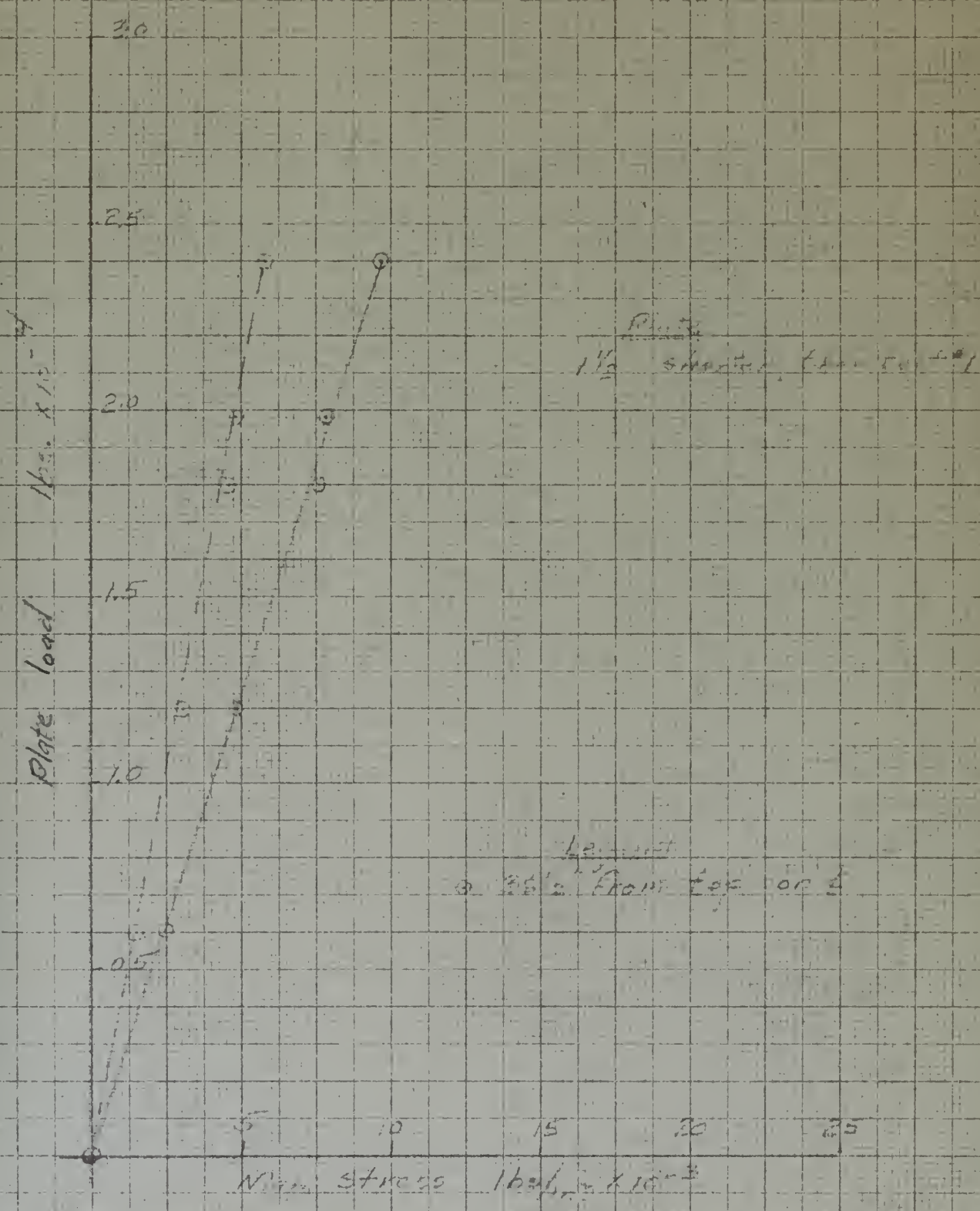


Fig. 1

Plate Load vs. Max. Cover (inches) - Test #21
 Ref. Table II

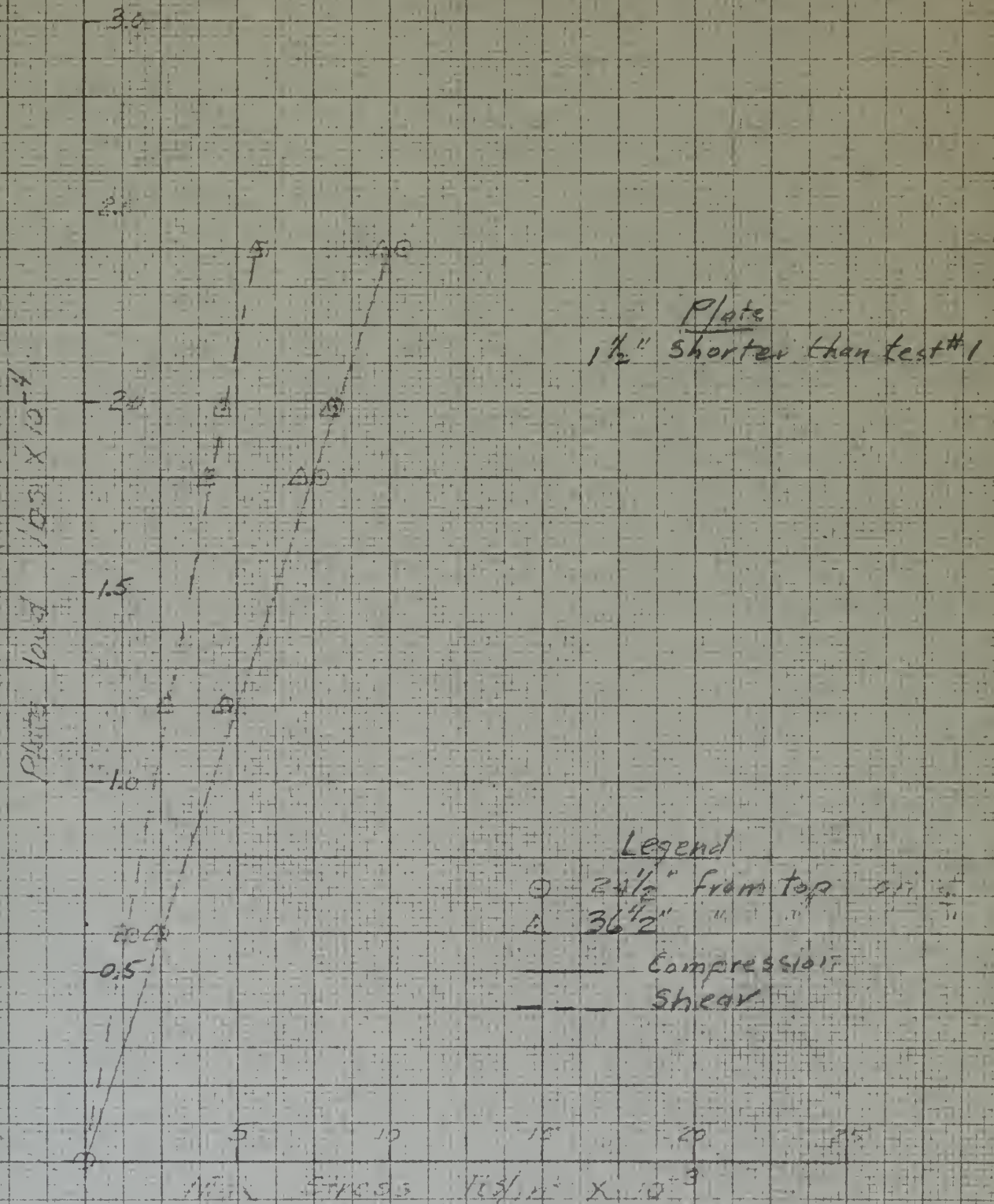


Fig. 10

Plate Load vs. Max. Corrugation Stress

TEST #2
Ref. Table 4.

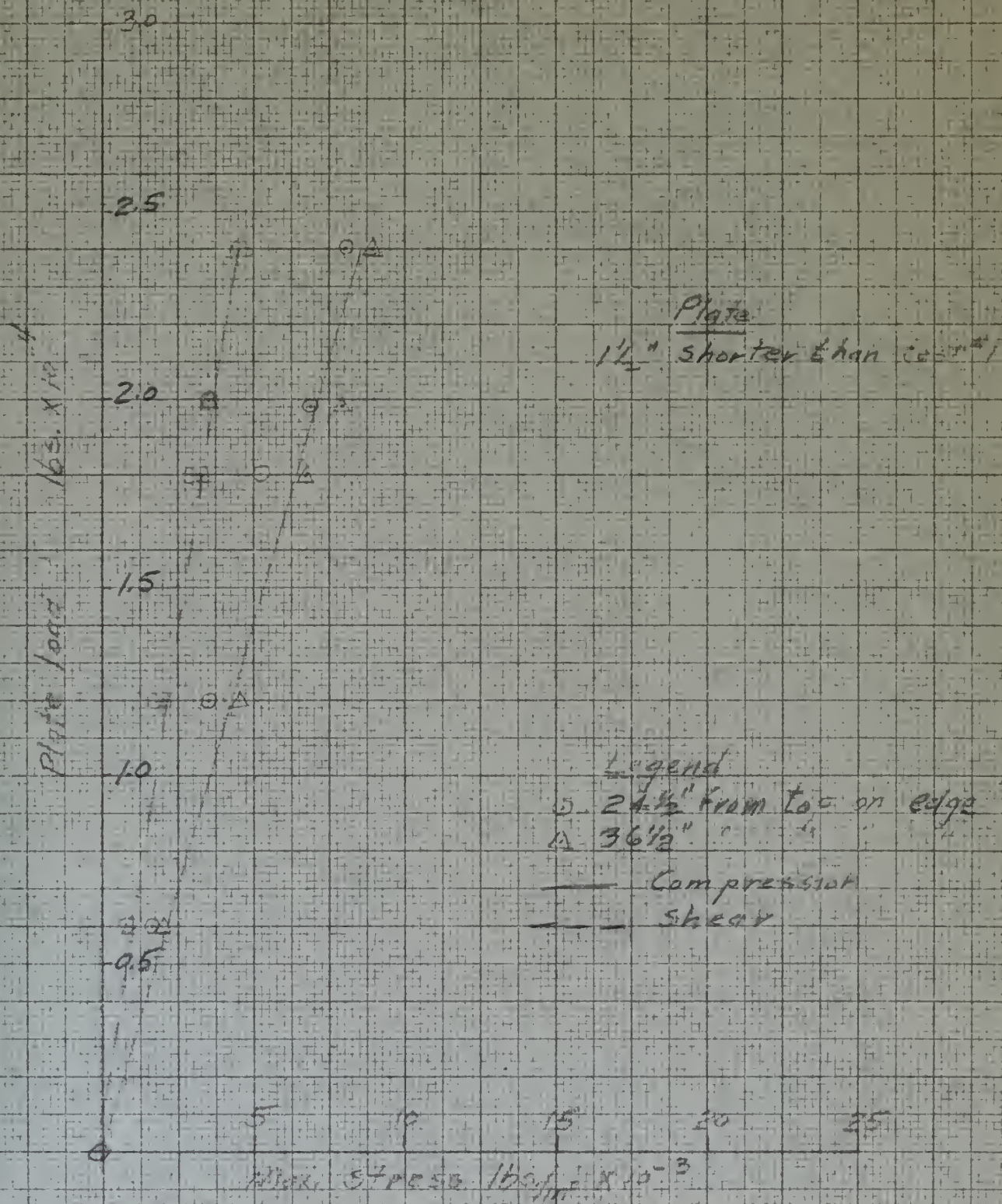


Fig. 11

Plate Load vs. Max. Cover Stress (N) Stress
(Test #3 Distribution)

Ref Table III

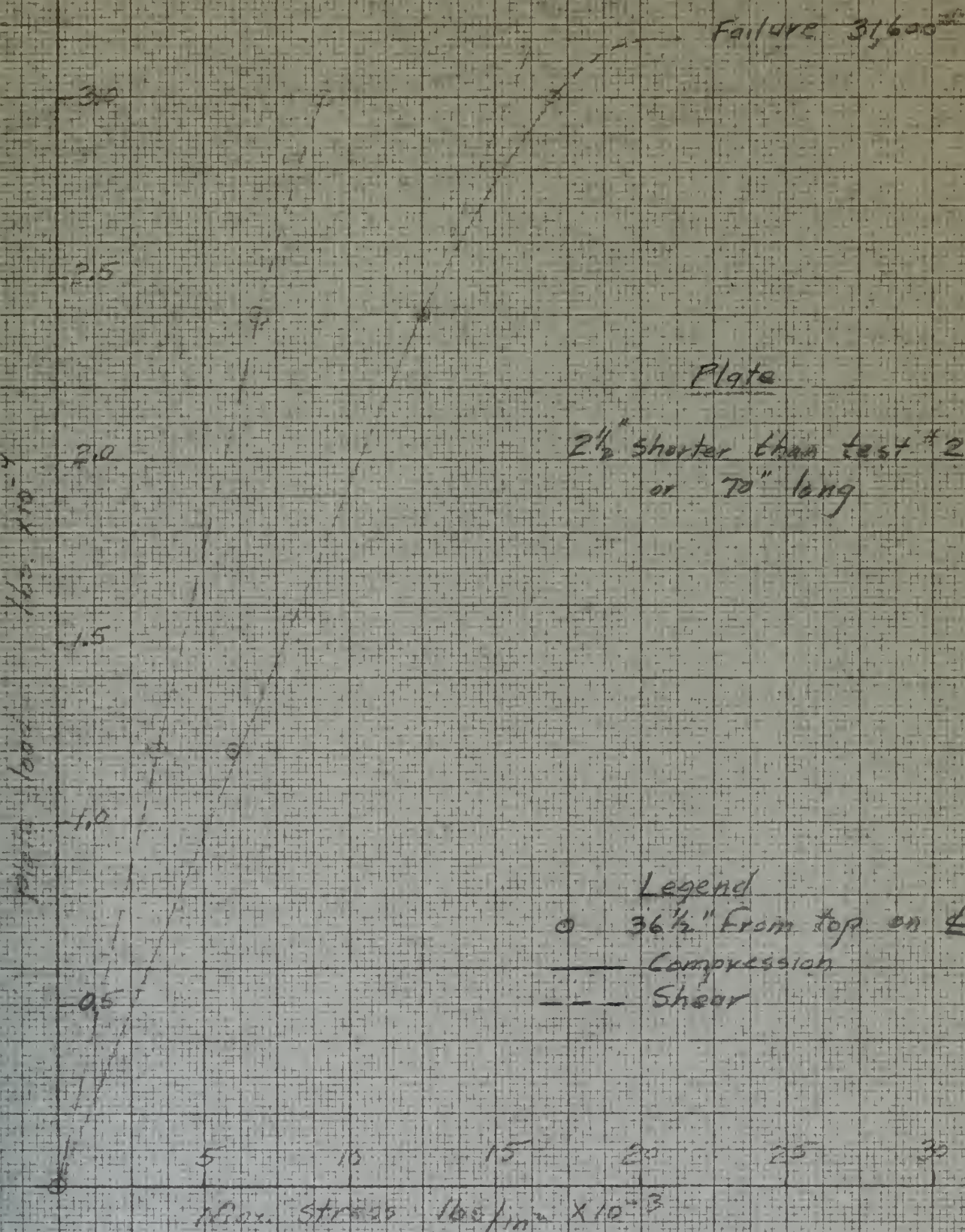


Fig. 12

Plate Load vs. Max. Cover Sheet(2) Stress
(Test #3 Destruction) Ref. Table II

Failure 31,600

Plate
2 1/2" shorter than test #2
L = 70"

Legend

○ 2 1/2" from top on #
△ 3 1/2" from top on #

— Compression
--- Shear

Plate Load

3.0

2.5

2.0

1.5

1.0

0.5

5

10

15

20

25

Max Stress lbs/in² x 10⁻³

Fig. 13

Plate Load vs. Max. Corrugation Stress
(Test #3) Ref. Table III

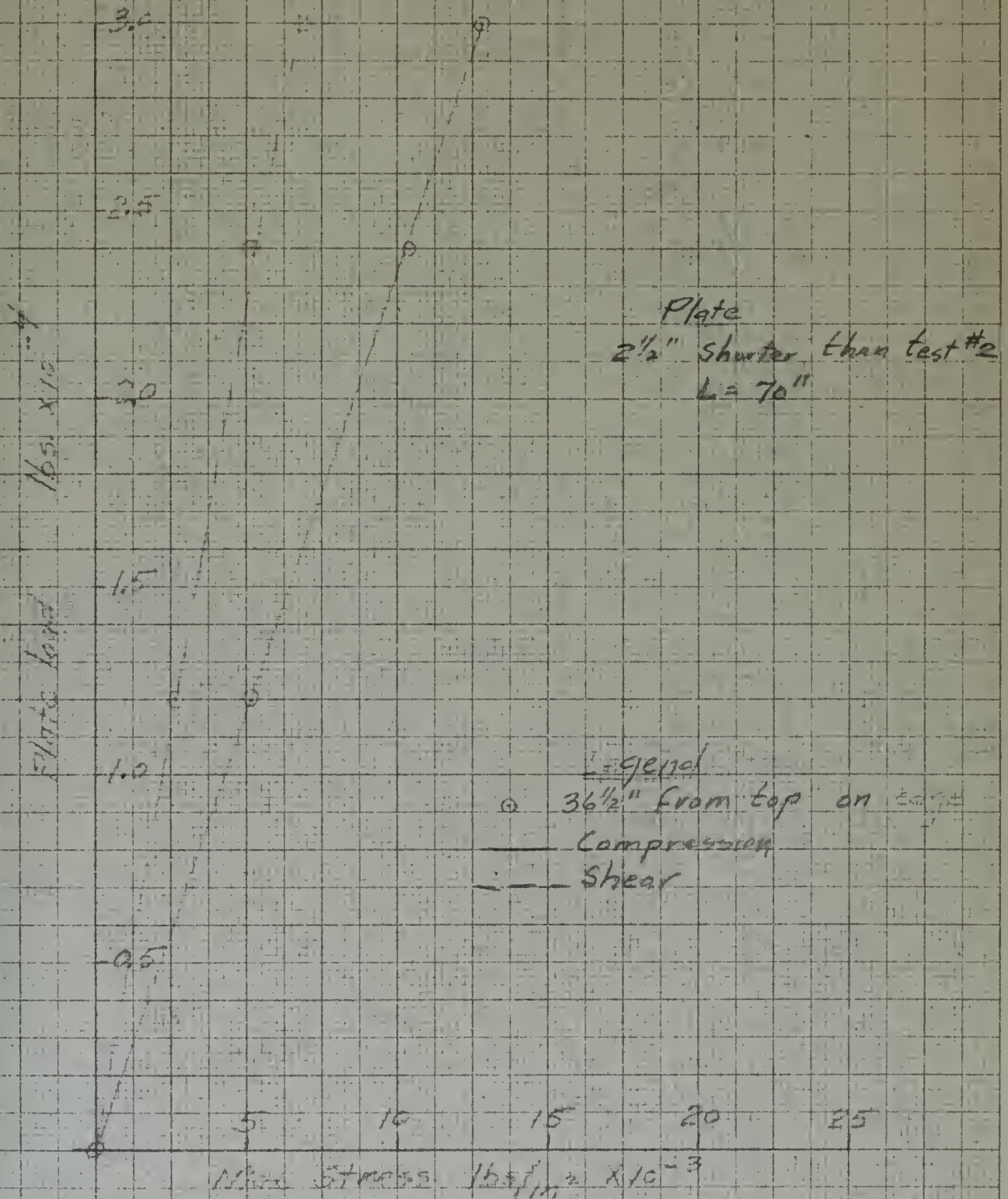


Fig. 14

Plate Load v.s. Maximum Stress
(Test # 4 Destruction)
Ref. Table II

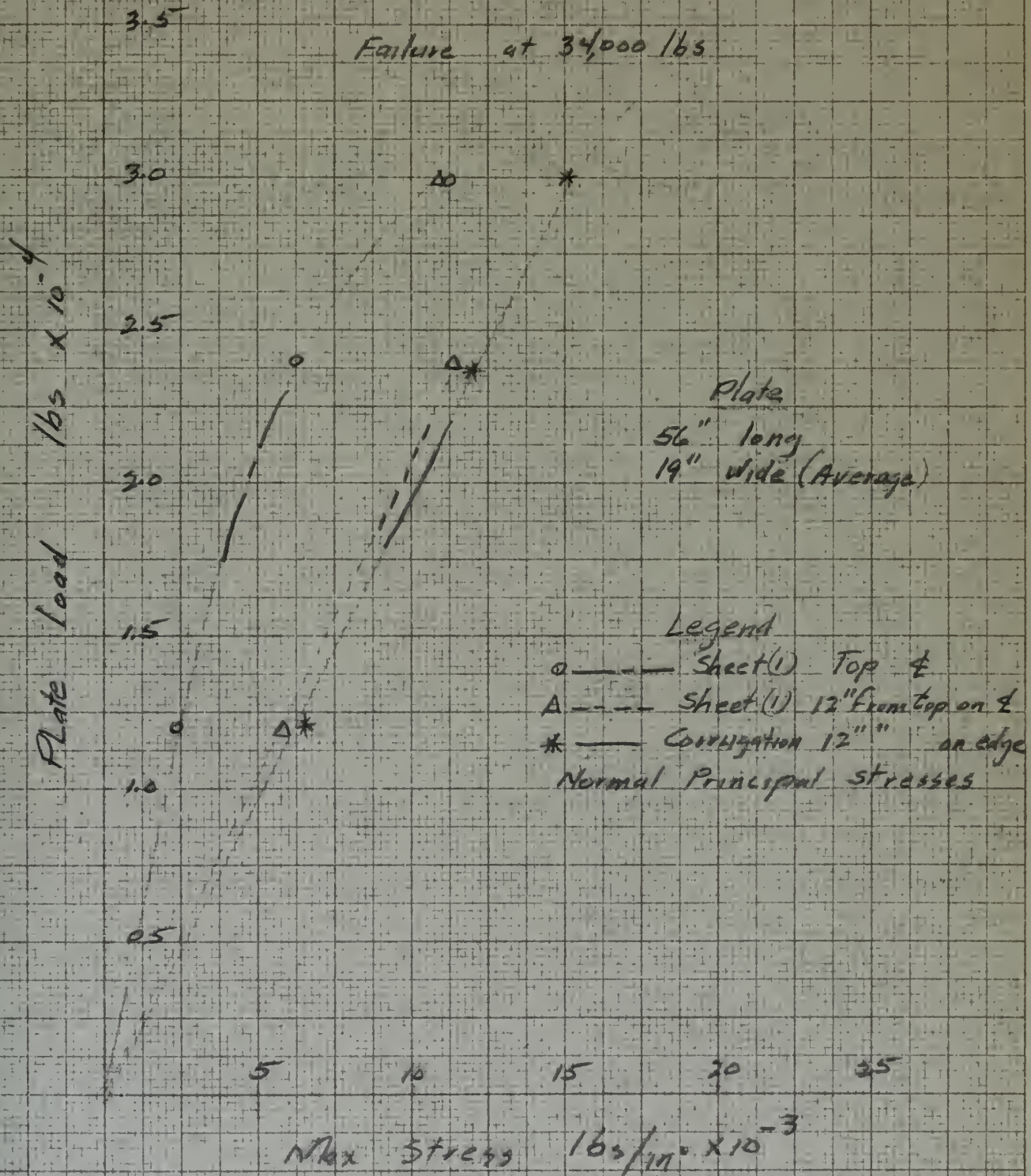
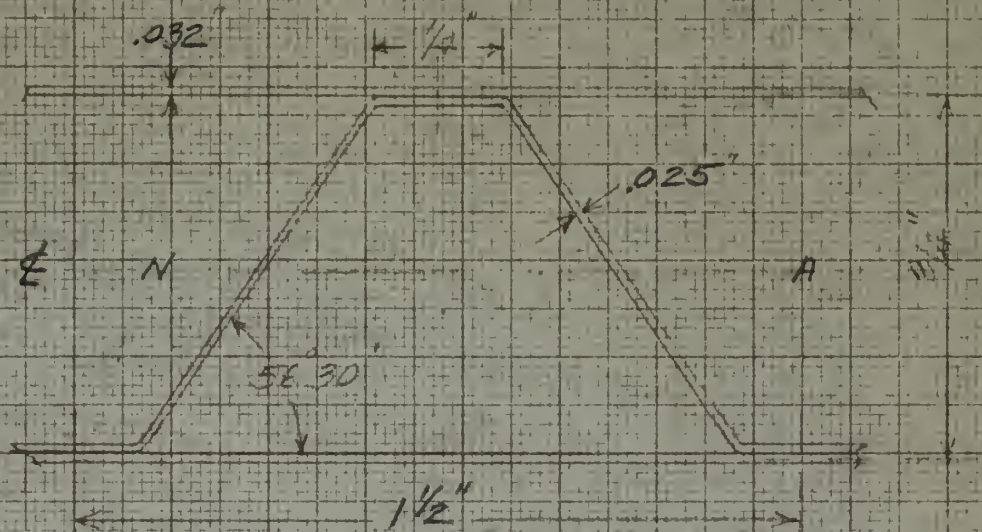


Fig. 15

Sheet and Corrugation Joint
 Properties
 (Average Section)



Sheet and Corrugation Joint
 by 1/8" cherry rivets
 on both sides

Properties

Sheet

$$A = 0.064 \text{ in.}^2/\text{in.}$$

$$I = 0.00165 \text{ in.}^4/\text{in.} \{ \text{bare properties} \}$$

$$14.5 \text{ ksi Alclad}$$

$$t_s = 0.0272 \text{ bare}$$

Corrugations

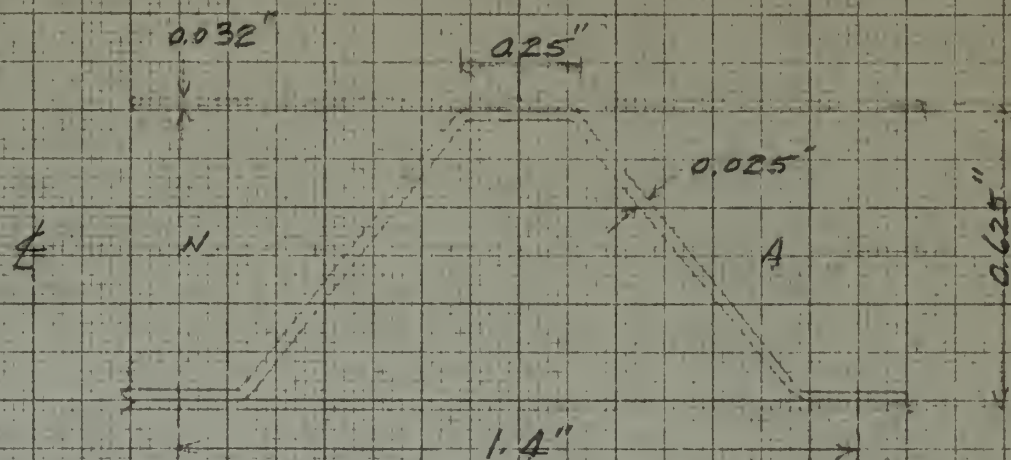
$$A = 0.0357 \text{ in.}^2/\text{in.}$$

$$I = 0.00245 \text{ in.}^4/\text{in.} \{ \text{bare properties} \}$$

$$30.1 \text{ ksi Alclad}$$

$$t_c = 0.0222 \text{ bare}$$

Fig. 16
Sheet and Corrugation
Properties
(Minimum Section)



Sheet and Corrugation joined
by $\frac{1}{8}$ " Cherry rivets
on both sides

Properties

Sheet

$A = 0.064 \text{ in}^2/\text{in}$
 $I = 0.005475 \text{ in}^4/\text{in}$ {bare properties}
 $t_s = 0.0272 \text{ in. bare}$
 145-T Alclad

Corrugation

$A = 0.0375 \text{ in}^2/\text{in}$
 $I = 0.00159 \text{ in}^4/\text{in}$
 $t_c = 0.0222 \text{ in. bare}$
 301-T Alclad

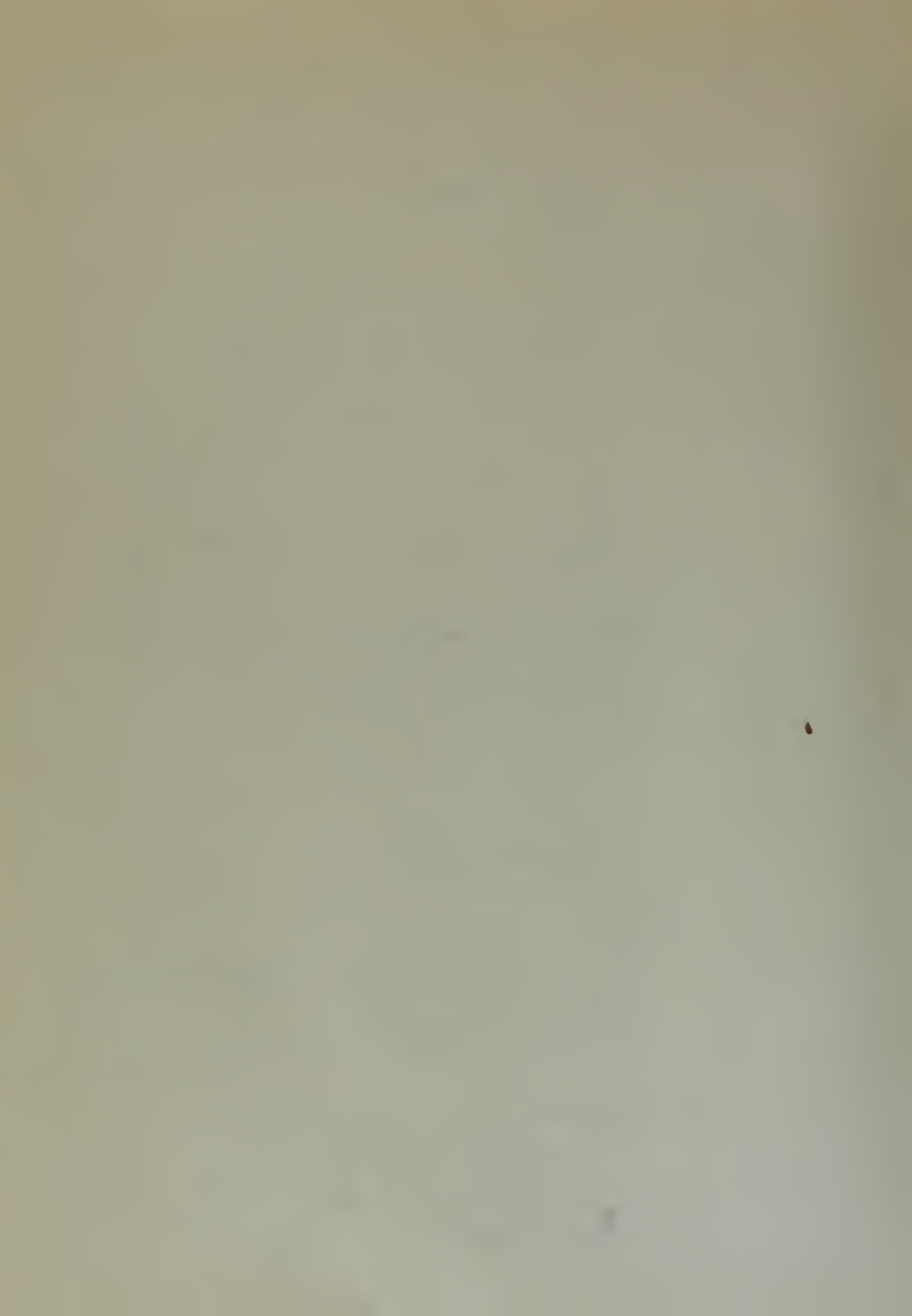


Fig. 17

Compression Properties 301-T, 14S-T
Alclad Sheet

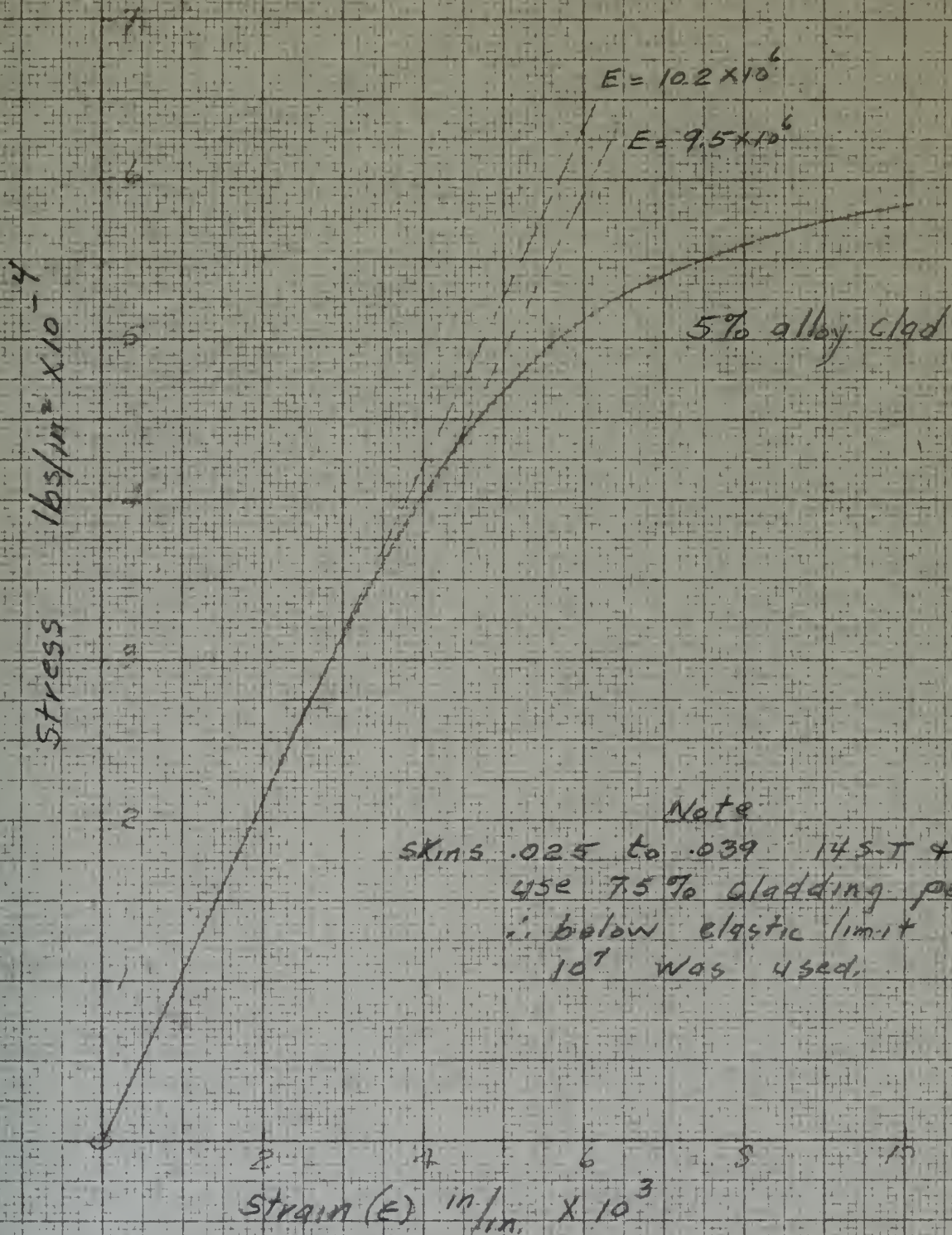


Fig. 18

Tangent Modulus of Elasticity
vs. Strain for 14 ST-3017 Alclad

Ref. Fig. 17

Note: When skins are below
0.39 % cladding is
greater than 5% per
side, slightly reducing E.

Tangent Modulus of Elasticity $10^6 \text{ lb/in}^2 \times 10^{-6}$

Strain $\times 10^3$

Fig. 19

Average stress of sheet vs. strain
beyond initial instability
point of sheet

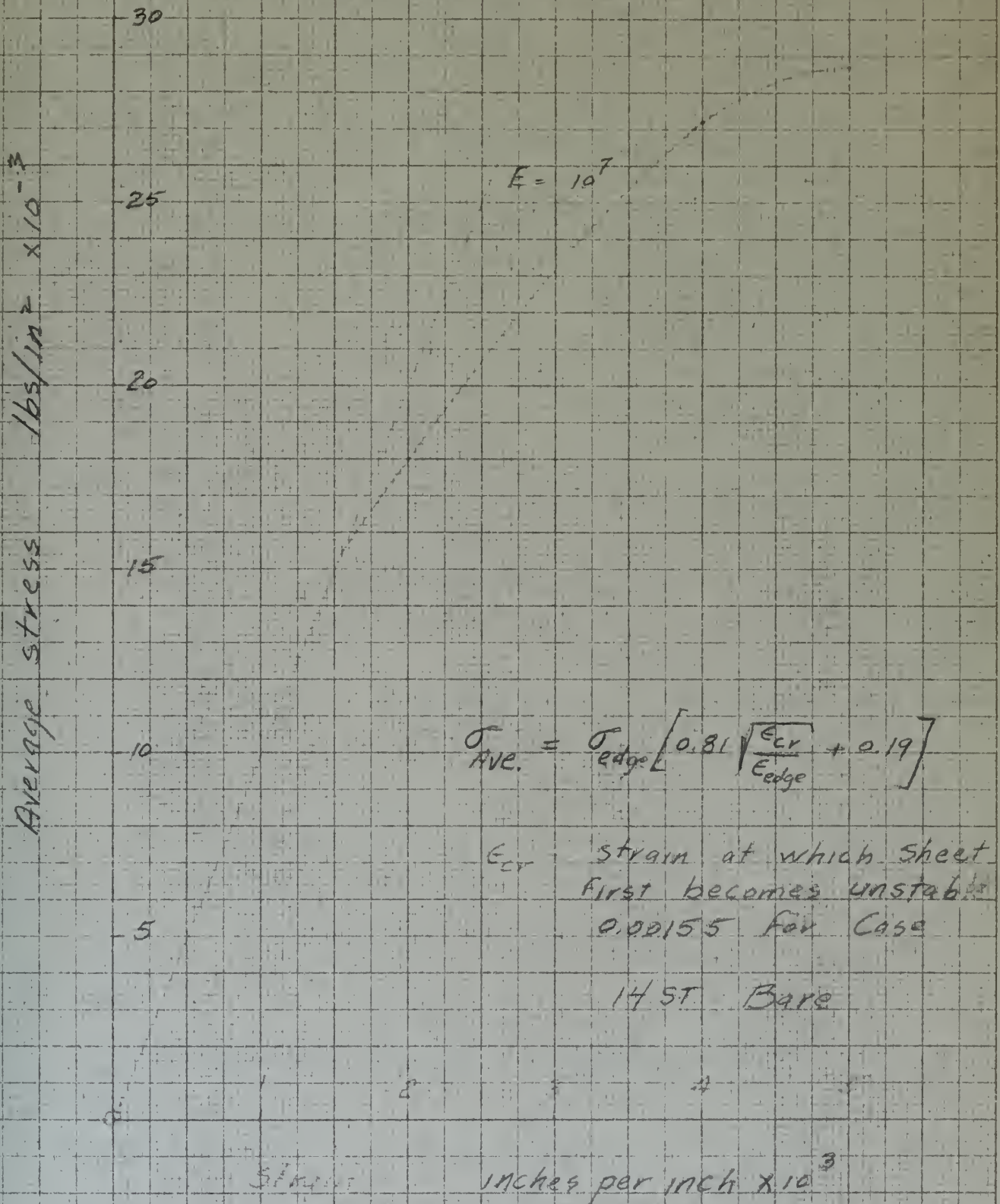


Fig. 2a

Tangent Modulus of Elasticity
vs. Strain For 14 ST sheet
beyond initial instability point
(Ref. Fig. 19)

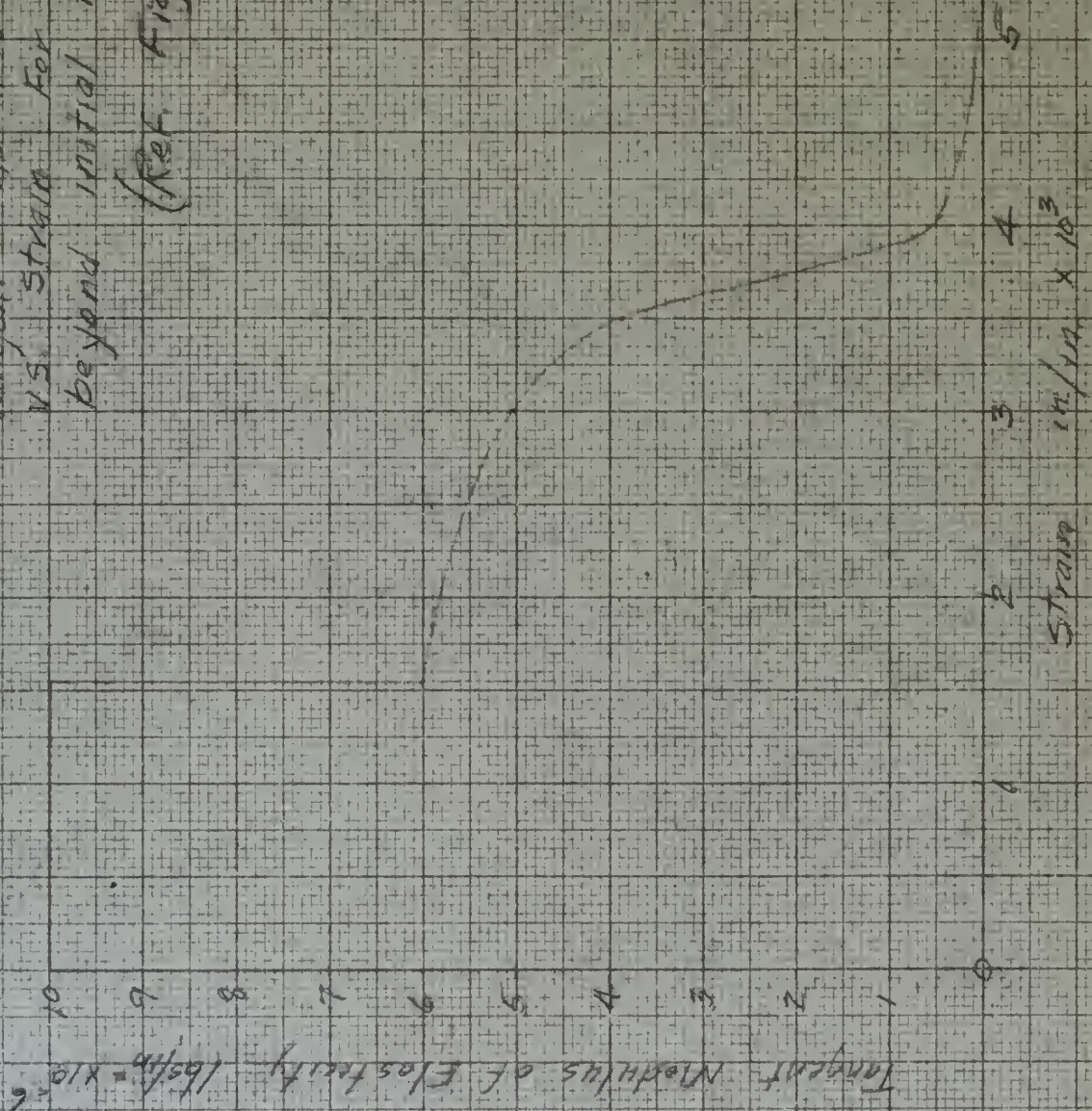
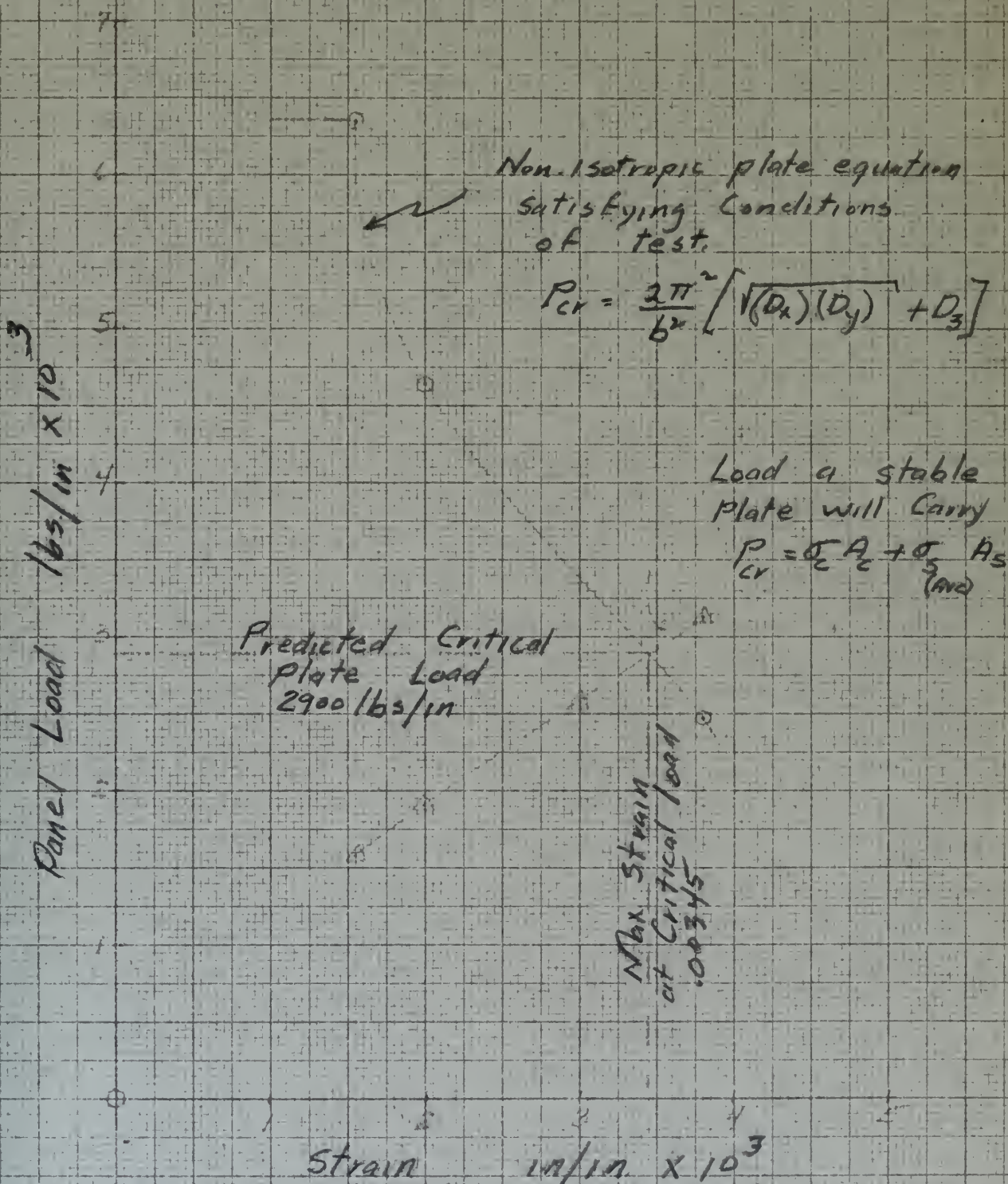
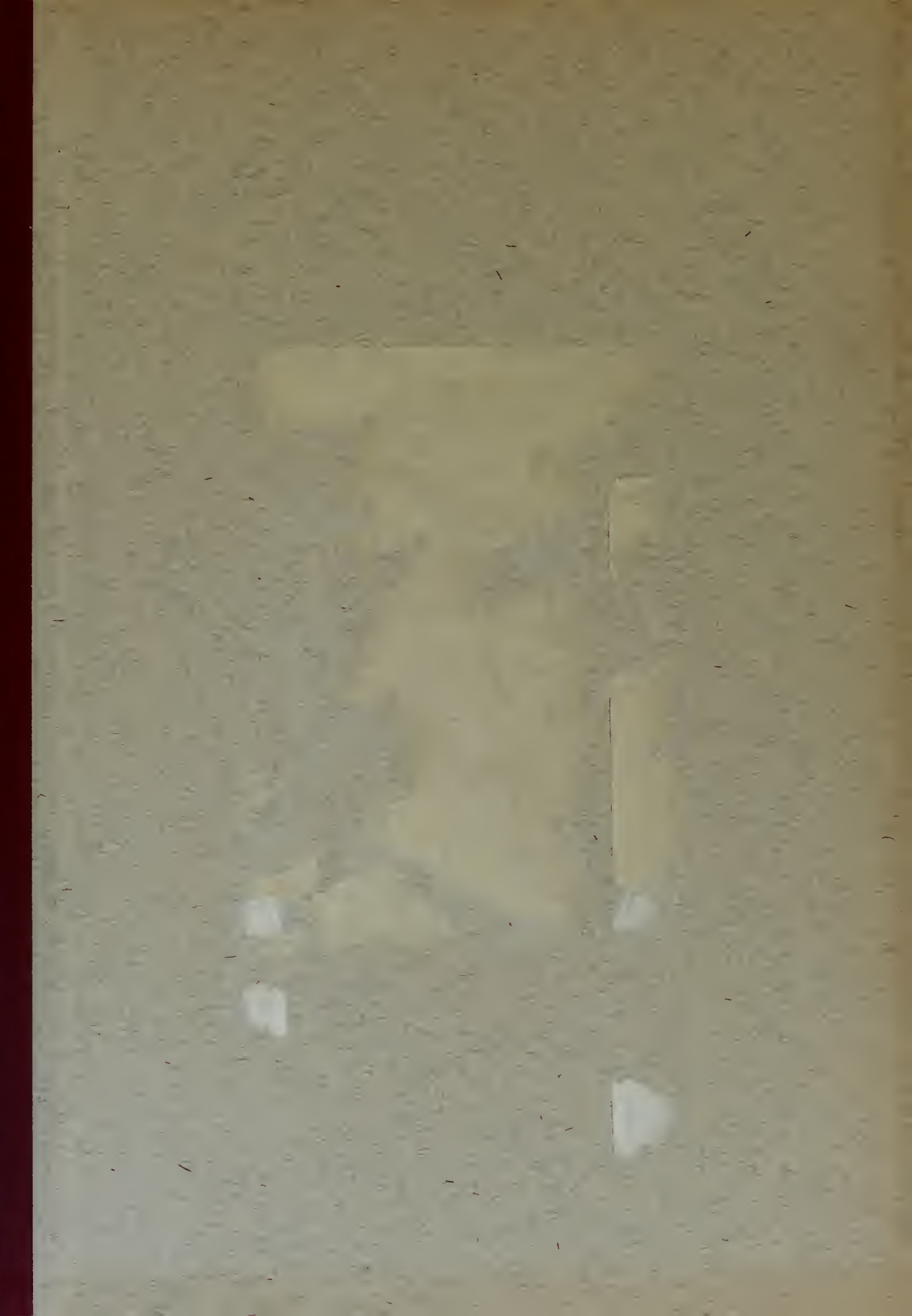


Fig. 21

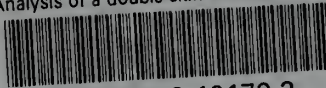
Predicted Critical Plate Load





thesi57

Analysis of a double skin and corrugated



3 2768 002 10179 2

DUDLEY KNOX LIBRARY